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# Non-parametric adaptive estimation of the drift for a jump diffusion process

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**Abstract**

In this article, we consider a jump diffusion process $(X_t)_{t\geq 0}$ observed at discrete times $t = 0, \Delta, \ldots, n\Delta$. The sampling interval $\Delta$ tends to 0 and $n\Delta$ tends to infinity. We assume that $(X_t)_{t\geq 0}$ is ergodic, strictly stationary and exponentially $\beta$-mixing. We use a penalised least-square approach to compute two adaptive estimators of the drift function $b$. We provide bounds for the risks of the two estimators.


*Keywords:* Jump diffusions; Nonparametric estimation; Drift estimation; Model selection

## 1. Introduction

We consider a general diffusion with jumps:

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t + \xi(X_{t^-})dL_t \quad \text{and} \quad X_0 = \eta \tag{1}$$

where $L_t$ is a centred pure jump Levy process:

$$dL_t = \int_{z\in\mathbb{R}} z\,(\mu(dt, dz) - dt\nu(dz))$$

with $\mu$ a random Poisson measure with intensity measure $\nu(dz)dt$ such that $\int_{z\in\mathbb{R}} z^2\nu(dz) < \infty$. The compensated Poisson measure $\tilde{\mu}$ is defined by $\tilde{\mu}(dt, dz) = \mu(dt, dz) - \nu(dz)dt$. The random variable $\eta$ is independent of $(W_t, L_t)_{t\geq 0}$. Moreover, $(W_t)_{t\geq 0}$ and $(L_t)_{t\geq 0}$ are independent.

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This process is observed with high frequency (at times $t = 0, \Delta, \ldots, n\Delta$ where, as $n$ tends to infinity, the sampling interval $\Delta \to 0$ and the time of observation $n\Delta \to \infty$). It is assumed to be ergodic, stationary and exponentially $\beta$-mixing (see [15] for sufficient conditions). Our aim is to construct a non-parametric estimator of $b$ on a compact set $A$.

The non-parametric estimation of $b$ and $\sigma$ for a diffusion process observed with high-frequency is well-known (see for instance [10,6]). Diffusion processes with jumps are used in various fields, for instance in finance, for modelling the growth of a population, in hydrology, in medical science, ..., but there exist few results for the non-parametric estimation of $b$ and $\sigma$. Mai [13] and Shimizu and Yoshida [21] construct maximum-likelihood estimators of parameters of $b$. Their estimators reach the standard rate of convergence, $\sqrt{n\Delta}$. Shimizu [20] and Mancini and Renò [14] use a kernel estimator to obtain non parametric threshold estimators of $\sigma$. Mancini and Renò [14] also construct a non-parametric truncated estimator of $b$, but only when $L_t$ is a compound Poisson process. To our knowledge, minimax rates of convergences for non-parametric estimators of $b$, $\sigma$ or $\xi$ for jump–diffusion processes are not available in the literature (see [10] or [9] for rates of convergence for diffusion processes).

In this paper, we use model selection to construct two non-parametric estimators of $b$ under the asymptotic framework $\Delta \to 0$ and $n\Delta \to \infty$. This method was introduced by Birgé and Massart [4].

First, we introduce a sequence of linear subspaces $S_m \subseteq L^2(A)$ and, for each $m$, we construct an estimator $\hat{b}_m$ of $b$ by minimising on $S_m$ the contrast function:

$$\gamma_n(t) = \frac{1}{n}\sum_{k=1}^{n}(Y_{k\Delta} - t(X_{k\Delta}))^2 \quad \text{where } Y_{k\Delta} = \frac{X_{(k+1)\Delta} - X_{k\Delta}}{\Delta}.$$

We obtain a collection of estimators of the drift function $b$ and we bound their risks (Theorem 2). Then, we introduce a penalty function to select the "best" dimension $m$ and we deduce an adaptive estimator $\hat{b}_{\hat{m}}$. Under the assumption that $\nu$ is sub-exponential, that is if there exist two positive constants $C, \lambda$ such that, for $z$ large enough, $\nu([-z, z]^c) \leq Ce^{-\lambda z}$, the risk bound of $\hat{b}_{\hat{m}}$ is exactly the same as for a diffusion without jumps (Theorem 4) (see [6] or [10]).

In a second part, we do not assume that $\nu$ is sub-exponential and we construct a truncated estimator $\tilde{b}_m$ of $b$. We minimise the contrast function

$$\tilde{\gamma}_n(t) = \frac{1}{n}\sum_{k=1}^{n}\left(Y_{k\Delta}\mathbb{1}_{|Y_{k\Delta}|\leq C_\Delta} - t(X_{k\Delta})\right)^2 \quad \text{where } C_\Delta \propto \sqrt{\Delta}\ln(n)$$

in order to obtain a new estimator $\tilde{b}_m$. As in the first part, we introduce a penalty function to obtain an adaptive estimator $\tilde{b}_{\tilde{m}}$. The risk bound of this adaptive estimator depends on the Blumenthal–Getoor index of $\nu$ (Theorems 7 and 10).

In Section 2, we present the model and its assumptions. In Sections 3 and 4, we construct the estimators and bound their risks. Some simulations are presented in Section 5. Proofs are gathered in Section 6.

## 2. Assumptions

### 2.1. Assumptions on the model

We consider the following assumptions.

**A1.** The functions $b$, $\sigma$ and $\xi$ are Lipschitz.