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Weak solutions of backward stochastic differential equations with continuous generator

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Abstract

We prove the existence of a weak solution to a backward stochastic differential equation (BSDE)

$$Y_t = \xi + \int_t^T f(s, X_s, Y_s, Z_s)\, ds - \int_t^T Z_s\, dW_s$$

in a finite-dimensional space, where $f(t, x, y, z)$ is affine with respect to z, and satisfies a sublinear growth condition and a continuity condition. This solution takes the form of a triplet (Y, Z, L) of processes defined on an extended probability space and satisfying

$$Y_t = \xi + \int_t^T f(s, X_s, Y_s, Z_s)\, ds - \int_t^T Z_s\, dW_s - (L_T - L_t)$$

where L is a martingale with possible jumps which is orthogonal to W. The solution is constructed on an extended probability space, using Young measures on the space of trajectories. One component of this space is the Skorokhod space $\mathbb{D}$ endowed with the topology S of Jakubowski.

MSC: 60H10

Keywords: Weak solution; Joint solution measure; Young measure; Jakubowski's topology S; Condition UT; Meyer–Zheng; Yamada–Watanabe–Engelbert

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1. Introduction

Aim of the paper. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathrm{P})$ be a complete probability space, where $(\mathcal{F}_t)_{t\geq 0}$ is the natural filtration of a standard Brownian motion $W = (W_t)_{t\in[0,T]}$ on $\mathbb{R}^m$ and $\mathcal{F} = \mathcal{F}_T$.

In this paper, we prove the existence of a weak solution (more precisely, a solution defined on an extended probability space) to the equation

$$Y_t = \xi + \int_t^T f(s, X_s, Y_s, Z_s)\, ds - \int_t^T Z_s\, dW_s - (L_T - L_t) \tag{1}$$

where $f(t, x, y, z)$ is affine with respect to z, and satisfies a sublinear growth condition and a continuity condition, W is an $\mathbb{R}^m$-valued standard Brownian motion, Y and Z and L are unknown processes, Y and L take their values in $\mathbb{R}^d$, Z takes its values in the space $\mathbb{L}$ of linear mappings from $\mathbb{R}^m$ to $\mathbb{R}^d$, $\xi \in \mathrm{L}^2_{\mathbb{R}^d}$ is the terminal condition, and L is a martingale orthogonal to W, with $L_0 = 0$ and with càdlàg trajectories (i.e. right continuous trajectories with left limits at every point). The process $X = (X_t)_{0\leq t\leq T}$ is $(\mathcal{F}_t)$-adapted and continuous with values in a separable metric space $\mathbb{M}$. This process represents the random part of the generator f and plays a very small role in our construction. The space $\mathbb{M}$ can be, for example, some space of trajectories, and X_t can be, for example, the history until time t of some process ζ, i.e. $X_t = (\zeta_{s\wedge t})_{0\leq s\leq T}$.

Such a weak solution to (1) can be considered as a generalized weak solution to the more classical equation

$$Y_t = \xi + \int_t^T f(s, X_s, Y_s, Z_s)\, ds - \int_t^T Z_s\, dW_s. \tag{2}$$

Historical comments. Existence and uniqueness of the solution (Y, Z) to a nonlinear BSDE of the form

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\, ds - \int_t^T Z_s\, dW_s$$

have been proved in the seminal paper [30] by E. Pardoux and S. Peng, in the case when the generator f is random with $f(\cdot, 0, 0) \in \mathrm{L}^2(\Omega \times [0, T])$, and $f(t, y, z)$ is Lipschitz with respect to (y, z), uniformly in the other variables. In [26], J.P. Lepeltier and J. San Martín proved in the one dimensional case the existence of a solution when f is random, continuous with respect to (y, z) and satisfies a linear growth condition $\|f(t, y, z)\| \leq C(1 + \|y\| + \|z\|)$.

Equations of the form (2), with f depending on some other process X, appear in forward–backward stochastic differential equations (FBSDEs), where X is a solution of a (forward) stochastic differential equation.

As in the case of stochastic differential equations, one might expect that BSDEs with continuous generator always admit at least a *weak solution*, that is, a solution defined on a different probability space (generally with a larger filtration than the original one). A work in this direction but for forward–backward stochastic differential equations (FBSDEs) is that of K. Bahlali, B. Mezerdi, M. N'zi and Y. Ouknine [4], where the original probability is changed using Girsanov's theorem. Let us also mention the works on weak solutions to FBSDEs by Antonelli and Ma [2], and Delarue and Guatteri [13], where the change of probability space comes from the construction of the forward component.

Weak solutions where the filtration is enlarged have been studied by R. Buckdahn, H.J. Engelbert and A. Răşcanu in [11] (see also [9,10]), using pseudopaths and the Meyer–Zheng