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Particle filters with random resampling times

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Abstract

Particle filters are numerical methods for approximating the solution of the filtering problem which use systems of weighted particles that (typically) evolve according to the law of the signal process. These methods involve a corrective/resampling procedure which eliminates the particles that become redundant and multiplies the ones that contribute most to the resulting approximation. The correction is applied at instances in time called resampling/correction times. Practitioners normally use certain overall characteristics of the approximating system of particles (such as the effective sample size of the system) to determine when to correct the system. As a result, the resampling times are random. However, in the continuous time framework, all existing convergence results apply only to particle filters with deterministic correction times. In this paper, we analyse (continuous time) particle filters where resampling takes place at times that form a sequence of (predictable) stopping times. We prove that, under very general conditions imposed on the sequence of resampling times, the corresponding particle filters converge. The conditions are verified when the resampling times are chosen in accordance to the effective sample size of the system of particles, the coefficient of variation of the particles' weights and, respectively, the (soft) maximum of the particles' weights. We also deduce central-limit theorem type results for the approximating particle system with random resampling times.

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1. Introduction

The filtering problem involves the estimation of the current state of an evolving dynamical system based on partial observation. The evolution of the dynamical system is customarily modelled by a stochastic process $X = \{X_t, t \geq 0\}$ called the *signal* process, where the temporal parameter t runs over the positive half line $[0, \infty)$. The signal process X cannot be measured directly. However, a partial measurement of the signal can be obtained. This measurement is modelled by another continuous time process $Y = \{Y_t, t \geq 0\}$ which is called the *observation* process. The observation process is a function of X and a measurement noise. The measurement noise is modelled by a stochastic process $W = \{W_t, t \geq 0\}$. Hence,

$$Y_t = f_t(X_t, W_t) \quad t \in [0, \infty).$$

Let $\mathcal{Y} = \{\mathcal{Y}_t, t \geq 0\}$ be the filtration generated by the observation process Y; namely, $\mathcal{Y}_t = \sigma(Y_s, s \in [0, t])$, for $t \geq 0$. Then the filtering problem consists in computing π_t, the conditional distribution of X_t given $\mathcal{Y}_t$. The process $\pi = \{\pi_t, t \geq 0\}$ is a $\mathcal{Y}_t$-adapted probability measure valued process, so that

$$\mathbb{E}[\varphi(X_t) \mid \mathcal{Y}_t] = \int \varphi(x)\pi_t(dx),$$

for all statistics φ for which both terms of the above identity make sense. Generally speaking, the filtering problem cannot be solved analytically: an explicit formula cannot be obtained for the conditional distribution π_t. This is not true only in specific cases such as the Kalman–Bucy filter and the Benes filter (see, e.g. Chapter 6 in [1]). Numerical methods, of which particle filters are an example, are thus employed to obtain approximations to the solution of the filtering problem.

Particle filters[1] are numerical methods that produce an approximation of π_t using empirical distributions of systems of evolving weighted particles. They are currently one of the most successful methods used to approximate the solution of the filtering problem (see [7] or Chapter VIII in [4] for an overview). The particles evolve according to the law of the signal process X and carry a weight proportional with the likelihood of their recent position/trajectory given the observation data. As time progresses, some of the weights diminish and so the corresponding particles essentially contribute less to the approximation process. In order to counter this phenomenon known as sample degeneracy, a correction procedure is introduced at particular times to cull the redundant particles and multiply the particles that contribute more significantly to the approximation process. This correction procedure is known as resampling and it was first introduced in the papers by Gordon et al. [10,11], Kitagawa [14]. These resampling/correction times are chosen in an adaptive manner and are usually determined by certain overall characteristics of the approximating particle system. One such characteristic (for which the results from below apply) is the effective sample size of the approximating particle system.

In the last fifteen years we have witnessed a rapid development of the theory of particle filters. The discrete time framework has been extensively studied and a multitude of convergence and stability results have been proved. A comprehensive description of these developments in the wider context of approximations of Feynman–Kac formulae can be found in Del Moral [5]. Results concerning particle filters for the continuous time filtering problem are far fewer than their discrete counterparts. For an up-to-date overview of these results, see Chapter VIII in [4].

[1] These methods are also known under the name of Sequential Monte Carlo Methods in the Statistics and the Engineering literature.

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