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Sharp critical behavior for pinning models in a random correlated environment

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Abstract

This article investigates the effect for random pinning models of long range power-law decaying correlations in the environment. For a particular type of environment based on a renewal construction, we are able to sharply describe the phase transition from the delocalized phase to the localized one, giving the critical exponent for the (quenched) free-energy, and proving that at the critical point the trajectories are fully delocalized. These results contrast with what happens both for the pure model (i.e., without disorder) and for the widely studied case of i.i.d. disorder, where the relevance or irrelevance of disorder on the critical properties is decided via the so-called Harris Criterion (Harris, 1974) [21].
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1. Introduction

1.1. Physical motivations

The effect of disorder long-range correlations on the critical properties of a physical system has been well-studied in the physics literature, historically in [31] for a general class of models, and in [30] for the phenomenon we are interested in: the adsorption of a polymer on a wall

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or a line. One example that arises in nature is the DNA sequence, that has been found [24,26] to exhibit long-range power-law correlations, and it is thought that some repetitive patterns are responsible for these correlations. It is of great interest to analyze how these correlations affect the DNA denaturation process. We study here a probabilistic model, that represents a polymer which is pinned on a line that presents strongly correlated disorder with repetitive (but not periodic) patterns, and we show that, according to physicists' predictions, the critical properties of the model are modified with respect to the case where the disorder is independent at each site of the line.

1.2. Definition of the model

Let $\tau := \{\tau_n\}_{n\geqslant 0}$ be a recurrent renewal sequence, that is a sequence of random variables such that $\tau_0 := 0$, and $\{\tau_{i+1} - \tau_i\}_{i\geqslant 0}$ are independent random variables identically distributed with support in $\mathbb{N}$, with common law (called inter-arrival distribution) denoted by $K(\cdot)$. The law of τ is denoted by $\mathbf{P}$. We assume that $K(\cdot)$ satisfies

$$K(n) := \mathbf{P}(\tau_1 = n) = (1 + o(1))\frac{c_K}{n^{1+\alpha}}, \tag{1.1}$$

for some $\alpha > 0, \alpha \neq 1$ (the assumption $\alpha \neq 1$ does not hide anything deep but it avoids various technical nuisances). The fact that the renewal is recurrent simply means that $K(\infty) = \mathbf{P}(\tau_1 = +\infty) = 0$. We assume also for simplicity that $K(n) > 0$ for all $n \in \mathbb{N}$. We use the notation

$$\bar{K}(n) := \mathbf{P}(\tau_1 > n) = \sum_{i=n+1}^{\infty} K(i). \tag{1.2}$$

With a slight abuse of notation, τ also denotes the set $\{k \in \mathbb{N} \mid \tau_n = k \text{ for some } n\}$. Given a sequence $\omega = (\omega_n)_{n\in\mathbb{N}}$ of real numbers (the environment), $h \in \mathbb{R}$ (the pinning parameter) and $\beta \geqslant 0$ (the inverse temperature), we define the sequence of polymer measures $\mathbf{P}_{N,h}^{\omega,\beta}$, $N \in \mathbb{N}$ as follows

$$\frac{\mathrm{d}\mathbf{P}_{N,h}^{\omega,\beta}}{\mathrm{d}\mathbf{P}}(\tau) := \frac{1}{Z_{N,h}^{\omega,\beta}} \exp\left(\sum_{n=1}^{N}(h + \beta\omega_n)\mathbf{1}_{\{n\in\tau\}}\right), \tag{1.3}$$

where

$$Z_{N,h}^{\omega,\beta} := \mathbf{E}\left[\exp\left(\sum_{n=1}^{N}(h + \beta\omega_n)\mathbf{1}_{\{n\in\tau\}}\right)\right] \tag{1.4}$$

is called the *partition function* of the system.

The set τ can be thought of as the set of return times to its departure point (call it 0) of some random walk S on some state space, say $\mathbb{Z}^d$. The graph of the random walk $(k, S_k)_{k\in[0,N]}$ is interpreted as a 1-dimensional polymer chain living in a $(d + 1)$-dimensional space, and interacting with the defect line $[0, N] \times \{0\}$. Physically, our modification of $\mathbf{P}$ corresponds to giving an energy reward (or penalty, depending on its sign) to the trajectory $(k, S_k)_{k\in[0,N]}$ when it touches the defect line, at the times $(\tau_i)_{i\in\mathbb{N}}$. The reward consists of an homogeneous part: h, and an inhomogeneous one: $\beta\omega_n$.

Our aim is to study the properties of $\tau \cap [0, N]$ under the polymer measure $\mathbf{P}_{N,h}^{\omega,\beta}$ for large values of N. This model, known as *inhomogeneous pinning model*, has been studied in depth

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