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Some fluctuation results for weakly interacting multi-type particle systems

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Abstract

A collection of N-diffusing interacting particles where each particle belongs to one of K different populations is considered. Evolution equation for a particle from population k depends on the K empirical measures of particle states corresponding to the various populations and the form of this dependence may change from one population to another. In addition, the drift coefficients in the particle evolution equations may depend on a factor that is common to all particles and which is described through the solution of a stochastic differential equation coupled, through the empirical measures, with the N-particle dynamics. We are interested in the asymptotic behavior as $N \to \infty$. Although the full system is not exchangeable, particles in the same population have an exchangeable distribution. Using this structure, one can prove using standard techniques a law of large numbers result and a propagation of chaos property. In the current work we study fluctuations about the law of large number limit. For the case where the common factor is absent the limit is given in terms of a Gaussian field whereas in the presence of a common factor it is characterized through a mixture of Gaussian distributions. We also obtain, as a corollary, new fluctuation results for disjoint sub-families of single type particle systems, i.e. when $K = 1$. Finally, we establish limit theorems for multi-type statistics of such weakly interacting particles, given in terms of multiple Wiener integrals.

MSC: 60F05; 60K35; 60H30; 60J70

Keywords: Mean field interaction; Common factor; Weakly interacting diffusions; Propagation of chaos; Central limit theorems; Fluctuation limits; Exchangeability; Symmetric statistics; Multiple Wiener integrals

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1. Introduction

For $N \geq 1$, let $Z^{1,N}, \ldots, Z^{N,N}$ be $\mathbb{R}^d$-valued stochastic processes, representing trajectories of N particles, each of which belongs to one of K types (populations) with the membership map denoted by $\boldsymbol{p} : \{1, \ldots, N\} \to \{1, \ldots, K\} \doteq \boldsymbol{K}$, namely ith particle is type α if $\boldsymbol{p}(i) = \alpha$. The dynamics is given in terms of a collection of stochastic differential equations (SDE) driven by mutually independent Brownian motions (BM) with each particle's initial condition governed independently by a probability law that depends only on its type. The N stochastic processes interact with each other through the coefficients of the SDE which, for the ith process, with $\boldsymbol{p}(i) = \alpha$, depend on not only the ith state process and the α-th type, but also the empirical measures $\mu_t^{\gamma,N} = \frac{1}{N_\gamma} \sum_{j:\boldsymbol{p}(j)=\gamma} \delta_{Z_t^{j,N}}$, $\gamma \in \{1, \ldots, K\}$ and a stochastic process that is common to all particle equations (common factor). Here N_γ is the total number of particles that belong to the γ-th type. The common factor is an m-dimensional stochastic process described once more through an SDE driven by a BM which is independent of the other noise processes (see equations (4.1)–(4.3)). Such stochastic systems are commonly referred to as *weakly interacting diffusion processes* and have a long history.

Classical works that study the law of large number (LLN) results and central limit theorems (CLT) include McKean [15,16], Braun and Hepp [2], Dawson [7], Tanaka [22], Oelschaläger [18], Sznitman [20,21], Graham and Méléard [10], Shiga and Tanaka [19], Méléard [17]. All the above papers consider exchangeable populations, i.e. $K = 1$, and a setting where the common factor is absent. Motivated by approximation schemes for Stochastic Partial Differential Equations (SPDE) the papers [13,14] considered a setting where the common factor is modeled as a Brownian sheet that drives the dynamics of each particle. The paper [13] studied LLN and [14] considered fluctuations about the LLN limit. In a setting where particle dynamics are given through jump–diffusions and the common factor is described by another jump–diffusion that is coupled with the particle dynamics, a CLT was recently obtained in [3]. The fluctuation limit theorems in [14,3], although allowing for a common factor, are limited to exchangeable populations. The goal of the current work is to study fluctuations for multi-type particle systems. Since these systems are not exchangeable (there is also no natural way to regard the system as a K-vector of d-dimensional exchangeable particles), classical techniques for proving CLT, developed in the above papers [20,19,17,14], are not directly applicable.

Multi-type systems have been proposed as models in social sciences [5], statistical mechanics [4], neurosciences [1], etc. In particular the last paper [1], considers interacting diffusions of the form studied in the current work and establishes a LLN result and a propagation of chaos property. Our results in particular will provide asymptotic results on fluctuations from the LLN behavior obtained in [1]. Systems with a common factor also arise in many different areas. In Mathematical Finance, they have been used to model correlations between default probabilities of multiple firms [6]; in neuroscience modeling these arise as systematic noise in the external current input to a neuronal ensemble [9]; and for particle approximation schemes for SPDE, the common factor corresponds to the underlying driving noise in the SPDE [13,14].

The goal of this work is to study a family of multi-type weakly interacting diffusions with a common factor. Our main objective is to establish a suitable CLT where the summands are quite general functionals of the trajectories of the particles with suitable integrability properties. Specifically, in the case where there is no common factor, letting $\boldsymbol{N_\alpha}$ denote the set of indices i