Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 126 (2016) 2297–2322

stochastic processes and their applications

www.elsevier.com/locate/spa

Drift operator in a viable expansion of information flow

Shiqi Song

Laboratoire Analyse et Probabilités, Université d'Evry Val D'Essonne, France

Received 8 July 2012; received in revised form 9 November 2015; accepted 1 February 2016
Available online 9 February 2016

Abstract

A triplet $(\mathbb{P}, \mathbb{F}, S)$ of a probability measure $\mathbb{P}$, of an information flow $\mathbb{F} = (\mathcal{F}_t)_{t\in\mathbb{R}_+}$, and of an $\mathbb{F}$ adapted asset process S, is a financial market model, only if it is viable. In this paper we are concerned with the preservation of the market viability, when the information flow $\mathbb{F}$ is replaced by a bigger one $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$ with $\mathcal{G}_t \supset \mathcal{F}_t$. Under the assumption of martingale representation property in $(\mathbb{P}, \mathbb{F})$, we prove a necessary and sufficient condition for all viable market in $\mathbb{F}$ remains viable in $\mathbb{G}$.

MSC: 60G07; 60G44; 60G40

Keywords: Enlargement of filtrations; Hypothesis(H'); Drift operator; Martingale representation property; Conditional multiplicity; Market viability; Structure condition; Local martingale deflator; No-arbitrage of first kind

1. Introduction

A financial market is modeled by a triplet $(\mathbb{P}, \mathbb{F}, S)$ of a probability measure $\mathbb{P}$, of an information flow $\mathbb{F} = (\mathcal{F}_t)_{t\in\mathbb{R}_+}$, and of an $\mathbb{F}$ adapted asset process S. The basic requirement about such a model is its viability. (The notion of viability has been defined in [14] for a general economy. It is now used more specifically to signify that the utility maximization problems have solutions, as in [9,29–33]. The viability is closely linked to the absences of arbitrage opportunity (of some kind) as explained in [33,28] so that the word sometimes is used to signify no-arbitrage condition.) There are situations where one should consider the asset process S in an enlarged information flow $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$ with $\mathcal{G}_t \supset \mathcal{F}_t$. The viability of the new market $(\mathbb{P}, \mathbb{G}, S)$ is

E-mail address: shiqi.song@univ-evry.fr.

http://dx.doi.org/10.1016/j.spa.2016.02.001
0304-4149/

not guaranteed. The purpose of this paper is to find such conditions that the viability will be maintained despite the expansion of the information flow.

Concretely, we introduce the notion of the full viability on a time horizon $[0, T]$ for information expansions (cf. Section 4.1 Definition 4.1). This means that, for any $\mathbb{F}$ special semimartingale asset process S, if $(\mathbb{P}, \mathbb{F}, S)$ is viable, the expansion market $(\mathbb{P}, \mathbb{G}, S)$ also is viable on $[0, T]$. Under the assumption of martingale representation property in $(\mathbb{P}, \mathbb{F})$, we prove that (cf. Theorem 4.3) the full viability on $[0, T]$ is equivalent to the following fact: there exist a (multi-dimensional) $\mathbb{G}$ predictable process φ and a (multi-dimensional) $\mathbb{F}$ local martingale N, such that (1) for any $\mathbb{F}$ local martingale X, the expression ${}^\top\varphi \centerdot [N, X]^{\mathbb{F}\cdot p}$ is well-defined on $[0, T]$ and $X - {}^\top\varphi \centerdot [N, X]^{\mathbb{F}\cdot p}$ is a $\mathbb{G}$ local martingale on $[0, T]$; (2) the continuous increasing process ${}^\top\varphi(\centerdot[N^c, {}^\top N^c])\varphi$ is finite on $[0, T]$; (3) the jump increasing process $(\sum_{0<s\le t} \left(\frac{{}^\top\varphi_s \Delta_s N}{1+{}^\top\varphi_s \Delta_s N}\right)^2)^{1/2}$, $t \in \mathbb{R}_+$, is $(\mathbb{P}, \mathbb{G})$ locally integrable on $[0, T]$.

It is to note that, if no jumps occurs in $\mathbb{F}$, continuous semimartingale calculus gives a quick solution to the viability problem of the information expansion. The situation becomes radically different when jumps occur, especially because we need to compute and to compare the different projections in $\mathbb{F}$ and in $\mathbb{G}$. (The problem is already difficult, even in the case where the filtration does not change. See [28,31].) In this paper we come to a satisfactory result in a general jump situation, thanks to a particular property derived from the martingale representation. In fact, when a process W has the martingale representation property, the jump ΔW of this process can only take a finite number of "predictable" values. We refer to [40] for a detailed account, where it is called the finite predictable constraint condition (which has a closed link with the notion of multiplicity introduced in [6]).

Usually the martingale representation property is mentioned to characterize a specific process (a Brownian motion, for example). But, in this paper, what is relevant is the stochastic basis $(\mathbb{P}, \mathbb{F})$ having a martingale representation property, whatever representation process is. One of the fundamental consequences of the finite predictable constraint condition is the possibility to find a finite family of very simply locally bounded mutually "avoiding" processes which have again the martingale representation property. This possibility reduces considerably the computation complexity and gives much clarity to delicate situations.

The viability property is fundamental for financial market modeling. There exists a huge literature (cf. for example, [9,11,12,16,17,27,28,31,30,33,37–39,41]). Recently, there is a particular attention on the viability problem related to expansions of information flow (cf. [1,2,13,42]). It is to notice that, however, the most of the works on expansions of information flow follow two specific ideas: the initial enlargement of filtration or the progressive enlargement of filtration (cf. [8,24,25,36,34] for definition and properties). In this paper, we take the problem in a very different perspective. We obtain general result, without assumption on the way that $\mathbb{G}$ is constructed from $\mathbb{F}$. It is known (cf. [43,44]) that the initial or progressive enlargement of filtration are particular situations covered by the so-called local solution method. The methodology of this paper does not take part in this category, adding a new element in the arsenal of filtration analysis.

The concept of information is a fascinating, but also a difficult notion, especially when we want to quantify it. The framework of enlargement of filtrations $\mathbb{F} \subset \mathbb{G}$ offers since long a nice laboratory to test the ideas. In general, no common consensus exists how to quantify the difference between two information flows $\mathbb{F}$ and $\mathbb{G}$. The notion of entropy has been used there (see for example [4,47]). But a more convincing measurement of information should be the drift operator $\Gamma(X)$, i.e. the operator which gives the drift part of the $\mathbb{F}$ local martingale X in $\mathbb{G}$