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Shuwen Lou, Cheng Ouyang

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FRACTAL DIMENSIONS OF ROUGH DIFFERENTIAL EQUATIONS DRIVEN BY FRACTIONAL BROWNIAN MOTIONS

SHUWEN LOU AND CHENG OUYANG

ABSTRACT. In this work we study fractal properties of a d-dimensional rough differential equation driven by fractional Brownian motions with Hurst parameter $H > \frac{1}{4}$. In particular, we show that the Hausdorff dimension of the sample paths of the solution is $\min\{d, \frac{1}{H}\}$ and that the Hausdorff dimension of the level set $L_x = \{t \in [\epsilon, 1] : X_t = x\}$ is $1 - dH$ with positive probability when $dH < 1$.

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1. INTRODUCTION

Random dynamical systems are well established modeling tools for a variety of natural phenomena ranging from physics (fundamental and phenomenological) to chemistry and more recently to biology, economics, engineering sciences and mathematical finance. In many interesting models the lack of any regularity of the external inputs of the differential equation as functions of time is a technical difficulty that hampers their mathematical analysis. The theory of rough paths has been initially developed by T. Lyons [17] in the 1990's to provide a framework to analyze a large class of driven differential equations and the precise relations between the driving signal and the output (that is the state, as function of time, of the controlled system).

Rough paths theory provides a nice framework to study differential equations driven by Gaussian processes (see [9]). In particular, using rough paths theory, we may define solutions of stochastic

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