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Multivalued backward stochastic differential equations with oblique subgradients[☆]

Anouar M. Gassous[a], Aurel Răşcanu[b,c], Eduard Rotenstein[b,*]

[a] Faculté des Sciences, Université Cadi Ayyad, Bd. Abdelkrim Khattabi, B.P. 511, Marrakech, Morocco
[b] Faculty of Mathematics, “Al. I. Cuza” University, Bd. Carol I, no. 9-11, Iaşi, Romania
[c] “Octav Mayer” Mathematics Institute of the Romanian Academy, Bd. Carol I, no. 8, Iaşi, Romania

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Abstract

We study the existence and uniqueness of the solution for the following backward stochastic variational inequality with oblique reflection (for short, *BSVI* $(H(t,y)\partial\varphi(y))$), written under differential form

$$\begin{cases} -dY_t + H\left(t, Y_t\right)\partial\varphi\left(Y_t\right)(dt) \ni F\left(t, Y_t, Z_t\right)dt - Z_t dB_t, \quad t \in [0, T], \\ Y_T = \eta, \end{cases}$$

where H is a bounded symmetric smooth matrix and φ is a proper convex lower semicontinuous function, with $\partial\varphi$ being its subdifferential operator. The presence of the product $H\partial\varphi$ does not permit the use of standard techniques because it conserves neither the Lipschitz property of the matrix nor the monotonicity property of the subdifferential operator. We prove that, if we consider the dependence of H only on the time, the equation admits a unique strong solution and, allowing the dependence on the state of the system, the above *BSVI* $(H(t,y)\partial\varphi(y))$ admits a weak solution in the sense of the Meyer–Zheng topology. However,

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* Corresponding author.
E-mail addresses: masigassous@gmail.com (A.M. Gassous), aurel.rascanu@uaic.ro (A. Răşcanu), eduard.rotenstein@uaic.ro (E. Rotenstein).

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for that purpose we must renounce at the dependence on Z for the generator function and we situate our problem in a Markovian framework.

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Keywords: Multivalued backward stochastic differential equations; Oblique reflection; Subdifferential operators; Meyer–Zheng topology

1. Introduction

Backward stochastic differential equations (BSDEs, in abbreviation) were introduced by Bismut in 1973 in the paper [2], as equation for the adjoint process in the stochastic version of Pontryagin maximum principle. In 1990, Pardoux and Peng [15] generalized and consecrated the well known now notion of nonlinear backward stochastic differential equation and they provided existence and uniqueness results for the solution of this equation. In [16], Pardoux and Peng developed a stochastic approach for proving the existence of a solution for deterministic partial differential equations. Since then, the interest in BSDEs has been growing, both in the direction of generalization of the emerging equations and on construction of approximation schemes for them. BSDEs have been widely used as a useful instrument for modeling various physical phenomena, in stochastic control or in mathematical finance, where, given a pricing problem, it can be written, by replication, in terms of a linear BSDE, or a non-linear BSDE with portfolios constraints. Pardoux and Răşcanu [17] proved, using a probabilistic interpretation, the existence of the viscosity solution for a multivalued PDE (with subdifferential operator) of parabolic and elliptic type.

Backward stochastic variational inequalities (BSVIs, for brevity) were first analyzed by Pardoux and Răşcanu in [17,18] by using a method that consisted of a penalizing scheme, followed by its convergence. Even though this type of penalization approach is very useful when dealing with multivalued backward stochastic dynamical systems governed by a subdifferential operator, it fails when dealing with a general maximal monotone operator. This motivated a new approach for such equations, via convex analysis tools. In [20], Răşcanu and Rotenstein established, using the Fitzpatrick function, a one-to-one correspondence between the solutions of those types of equations and the minimum points of some proper, convex, lower semicontinuous functions, defined on well-chosen Banach spaces.

Multi-dimensional BSDEs with oblique reflection (in fact BSDEs reflected on the boundary of a special unbounded convex domain along an oblique direction), which arise naturally in the study of optimal switching problem were recently studied by Hu and Tang in [7]. As applications, the authors used their results for solving the optimal switching problem for stochastic differential equations of functional type and they also gave a probabilistic interpretation of the viscosity solution for a system of variational inequalities.

It is worth mentioning that, until now, even for quite complex problems like the ones analyzed by Maticiuc and Răşcanu in [12] or [13], when dealing with BSVIs, the reflection was made upon the normal direction at the frontier of the domain and it was caused by the presence of the subdifferential operator of a convex lower semicontinuous function. As the main achievement of this paper we prove the existence and uniqueness of the solution for the more general BSVI with oblique subgradients