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Strong convergence in averaging principle for stochastic hyperbolic–parabolic equations with two time-scales

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Abstract

This article deals with averaging principle for stochastic hyperbolic–parabolic equations with slow and fast time-scales. Under suitable conditions, the existence of an averaging equation eliminating the fast variable for this coupled system is proved. As a consequence, an effective dynamics for slow variable which takes the form of stochastic wave equation is derived. Also, the rate of strong convergence for the slow component towards the solution of the averaging equation is obtained as a byproduct.

MSC: primary 60H15; secondary 70K70

Keywords: Stochastic hyperbolic–parabolic equations; Averaging principle; Invariant measure and ergodicity; Strong convergence

1. Introduction

Let $D = (0, L) \subset \mathbb{R}$ be a bounded open interval. In the article, for fixed $T_0 > 0$, we are concerned with the following stochastic hyperbolic–parabolic equation,

$$\frac{\partial^2 X_t^\epsilon(\xi)}{\partial t^2} = \Delta X_t^\epsilon(\xi) + f(X_t^\epsilon(\xi), Y_t^\epsilon(\xi)) + \sigma_1 \dot{W}_t^1(\xi), \tag{1.1}$$

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$$\frac{\partial Y_t^\epsilon(\xi)}{\partial t} = \frac{1}{\epsilon}\Delta Y_t^\epsilon(\xi) + \frac{1}{\epsilon} g(X_t^\epsilon(\xi), Y_t^\epsilon(\xi)) + \frac{\sigma_2}{\sqrt{\epsilon}}\dot{W}_t^2(\xi), \tag{1.2}$$

$$X_t^\epsilon(\xi) = Y_t^\epsilon(\xi) = 0, (\xi, t) \in \partial D \times (0, T_0], \tag{1.3}$$

$$X_0^\epsilon(\xi) = X_0(\xi), Y_0^\epsilon(\xi) = Y_0(\xi), \frac{\partial X_t^\epsilon(\xi)}{\partial t}|_{t=0} = \dot{X}_0(\xi), \xi \in D, \tag{1.4}$$

where the space variable $\xi \in D$, the time $t \in [0, T_0]$. The drifts f and g are suitable real-valued functions defined on $\mathbb{R}^2$ which are assumed to be Lipschitz continuous and in particular to have sublinear growth. The stochastic perturbations are of additive type and $W_t^1(\xi)$ and $W_t^2(\xi)$ are mutually independent Wiener processes on a complete stochastic basis $(\Omega, \mathscr{F}, \mathscr{F}_t, \mathbb{P})$, which will be specified later. The noise coefficients σ_1 and σ_1 are positive constants and the parameter ϵ is small and positive, which describes the ratio of time scale between the process $X_t^\epsilon(\xi)$ and $Y_t^\epsilon(\xi)$. With this time scale the variable $X_t^\epsilon(\xi)$ is referred as slow component and $Y_t^\epsilon(\xi)$ as the fast component.

The system (1.1)–(1.4) is an abstract model for a random vibration of a elastic string with external force on a large time scale. More generally, the nonlinear coupled wave–heat equations with fast and slow time scales may describe a thermoelastic wave propagation in a random medium [8], the interactions of fluid motion with other forms of waves [21,34], wave phenomena which are heat generating or temperature related [20], magneto-elasticity [24] and biological problems [7,3,28]. In this respect, the question of how the physical effects at large time scales influence the dynamics of the system (1.1)–(1.4) is arisen. We focus on this question and show that, under some dissipative conditions on fast variable equation (1.2), the complexities effects at large time scales to the asymptotic behavior of the slow component can be omitted or neglected in some sense.

Averaging methods are essential for describing and understanding the asymptotic behavior of dynamical systems with fast and slow variables. Its basic idea is to approximate the original system by a reduced system. The theory of averaging for deterministic dynamical systems, which was first studied by Bogoliubov [1], has a long and rich history. Further developments of the theory, for finite dimensional dynamical systems under random influences, was first shown by Khasminskii [15]. Since then, there is an extensive literature on this topic for finite dimensional systems with random perturbation (see Freidlin and Wentzell [11,12], Veretennikov [25,26] and Kifer [17–19]).

Recently, the averaging approach is developed to study the effective approximation to slow–fast random dynamical systems in infinite dimension. In [6] Cerrai and Freidlin presents an averaged result for stochastic parabolic equations with additive noise and Cerrai [4] deals with the case of multiplicative noise. The two papers show that the averaging principle, in sense of convergence in probability without a explicit rate, holds for that stochastic systems with fast and slow timescales. In [2] Bréhier derives explicit convergence rates in strong and weak convergence for averaging of stochastic parabolic equations when the additive noise is included only in the fast motion. These convergence rates are the same as for the finite dimensional cases [22,16].

However, to the best of our knowledge, the averaging principle for the stochastic hyperbolic–parabolic equations has not been so far solved. In this article, the main objective is to establish an effective approximation for slow process X_t^ϵ with respect to the limit $\epsilon \to 0^+$. As a consequence, a reduction system without the fast motion Y_t^ϵ, capturing the dynamics of the slow motion is derived. The averaging methods procedure can be used, as it is done in [6,4,5] for stochastic partial equations of parabolic type and for stochastic ordinary differential equations [14,22,29, 27,32,30,33,31]. To be more precise, the slow component X_t^ϵ can be approximated by the solution