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Hedging of defaultable claims in a structural model using a locally risk-minimizing approach

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Abstract

In the context of a locally risk-minimizing approach, the problem of hedging defaultable claims and their Föllmer–Schweizer decompositions are discussed in a structural model. This is done when the underlying process is a finite variation Lévy process and the claims pay a predetermined payout at maturity, contingent on no prior default. More precisely, in this particular framework, the locally risk-minimizing approach is carried out when the underlying process has jumps, the derivative is linked to a default event, and the probability measure is not necessarily risk-neutral.

Keywords: Defaultable claims; Hedging strategy; Locally risk-minimizing; Föllmer–Schweizer decomposition; Galtchouk–Kunita–Watanabe decomposition

1. Introduction

In its simple form, a defaultable claim pays a certain pre-defined amount at the maturity of the contract, if there has not been a prior default, and pays zero otherwise. In this work, a hedging analysis is carried out for these derivatives when the underlying risky asset is modeled by a

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finite variation Lévy process. It is of mathematical and practical interest to study the hedging of defaultable claims when the asset prices are affected by jumps. The extension to more complicated derivatives and underlying processes will be interesting for future work. First, we review the literature and related previous works.

We start by a definition of credit risk. Credit risk is the risk associated with the possible financial losses of a derivative caused by unexpected changes in the credit quality of the counterparty's issuer to meet its obligations. The first paper that introduced credit risk for a path independent claim goes back to the work of Merton [15].

When analyzing a credit derivative, normally there are two prominent issues, pricing and hedging of the derivative. The latter is a more challenging question, especially when the market is incomplete. In most financial models, even when working with simple stochastic processes, a complete hedge still may not be feasible for credit derivatives. There are different approaches to manage the risk in an incomplete market. Quadratic hedging is a well developed and applicable method to manage the risk.

Schweizer [25] or Pham [18] provide a good survey of quadratic hedging methods in incomplete markets. In [25] two quadratic hedging approaches are discussed for the case where the firm's value process is a semimartingale. These are local risk-minimization and mean–variance hedging.

If we prefer a self-financing portfolio in order to hedge a contingent claim, we speak of mean–variance hedging. If we rather select a portfolio with the same terminal value as the contingent claim (but not necessarily self-financing), we are in the context of a (locally) risk-minimizing approach. Schweizer, Heath and Platen [26] provide a comprehensive study and comparison of both approaches. In our paper a local risk-minimization approach is used to manage the risk associated with the defaultable claims.

Local risk-minimization hedging emerged in the development of the concept of risk minimization. Föllmer and Sondermann [8] were among the first to deal with this problem. They solved the problem identifying the risk-minimization strategy when the underlying process is a martingale. The generalization to the local martingale case is done in [25]. The solution of the risk-minimization problem is linked to the so called Galtchouk–Kunita–Watanabe (GKW) decomposition assuming that the underlying process is a local martingale.

For a non-martingale process, Schweizer [22] provides an example of an attainable claim that does not admit a risk-minimization strategy. The extension is possible by putting more restrictive conditions on the underlying process as well as on the hedging strategies.

Literally saying, one has to pay more attention to the local properties of the problem. As for the role of the underlying process, it has to satisfy the structure condition[1] (SC), see [23] or [25]. Under certain conditions like SC, a locally risk-minimizing strategy is equivalent to a more tractable one, called pseudo-locally risk-minimizing strategy. Föllmer and Schweizer [7] gives a necessary and sufficient condition for the existence of a pseudo-locally risk-minimizing strategy. It turns out that finding these strategies is equivalent to the existence of a generalized version of the GKW decomposition, known as the Föllmer–Schweizer (FS) decomposition. A sufficient condition for the existence of an FS decomposition is provided by Monat and Stricker [17].

[1] Assume that X is a square integrable special semimartingale with the canonical decomposition $X = X_0 + M + A$. Then X satisfies the structure condition, if there exists a predictable process λ such that $A_t = \int_0^t \lambda_s d\langle M\rangle_s$ for all $0 \le t \le T$, and the mean–variance tradeoff process, defined by $K_t = \int_0^t \lambda_s^2 d\langle M\rangle_s$, is $\mathbb{P}$-almost surely finite for all $0 \le t \le T$.