SPA: 2590 Model 1 pp. 1–25 (col. fig: NIL)

ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications xx (xxxx) xxx–xxx

stochastic processes and their applications

www.elsevier.com/locate/spa

Backward SDEs driven by Gaussian processes

Q1 Christian Bender*

Saarland University, Department of Mathematics, Postfach 151150, D-66041 Saarbrücken, Germany

Received 4 June 2013; received in revised form 26 March 2014; accepted 30 March 2014

Abstract

In this paper we discuss existence and uniqueness results for BSDEs driven by centered Gaussian processes. Compared to the existing literature on Gaussian BSDEs, which mainly treats fractional Brownian motion with Hurst parameter $H > 1/2$, our main contributions are: (i) Our results cover a wide class of Gaussian processes as driving processes including fractional Brownian motion with arbitrary Hurst parameter $H \in (0, 1)$; (ii) the assumptions on the generator f are mild and include e.g. the case when f has (super-)quadratic growth in z; (iii) the proofs are based on transferring the problem to an auxiliary BSDE driven by a Brownian motion.

MSC: 60H10; 60G15; 60G22; 60H05

Keywords: Backward SDEs; Fractional Brownian motion; Gaussian processes; Wick–Itô integration

1. Introduction

The theory of nonlinear backward stochastic differential equations (BSDEs) was initiated in the seminal paper by Pardoux and Peng [25] and is a vibrant field of research since then. When the driving process of the BSDE is a Brownian motion, there is by now a wealth of existence and uniqueness results under various assumptions on the coefficients. Besides the standard case of a Lipschitz continuous generator, for which we refer to the original paper by Pardoux and Peng [25] and the excellent survey paper by El Karoui et al. [13], we mention BSDEs with quadratic growth

* Tel.: +49 681 3022435.
E-mail address: bender@math.uni-sb.de.

http://dx.doi.org/10.1016/j.spa.2014.03.013
0304-4149/© 2014 Published by Elsevier B.V.

Please cite this article in press as: C. Bender, Backward SDEs driven by Gaussian processes, Stochastic Processes and their Applications (2014), http://dx.doi.org/10.1016/j.spa.2014.03.013

in the control part (usually denoted by Z) of the solution, which are of particular importance for applications in mathematical finance and optimal control. These so-called quadratic BSDEs were first studied by Kobylanski [21]. More generally, BSDEs with (super-)quadratic growth were recently investigated by Delbaen et al. [10] and Richou [28].

In this paper we are concerned with BSDEs driven by a class of centered Gaussian processes which includes fractional Brownian motion for the full range of Hurst parameters $H \in (0, 1)$. Concerning the driving centered Gaussian process X we assume that it has a strictly increasing continuous variance function $V(t) := \mathrm{Var}(X_t)$ and additionally impose some assumptions on the associated Cameron–Martin space, which can be verified e.g. in the fractional Brownian motion case. The Gaussian BSDE is then of the form

$$dY_t = -f(t, X_t, Y_t, Z_t)dV(t) + Z_t d^\diamond X_t, \qquad Y_T = g(X_T), \tag{1}$$

for deterministic coefficient functions f and g. In this equation the diamond indicates that integration with respect to X is defined in terms of the Wick product. We recall that there are several approaches to the Wick–Itô integral for Gaussian processes with a particular emphasis on the fractional Brownian motion case. Duncan et al. [11] construct this integral as the L^2-limit of Riemann sums which are based on the Wick product instead of the ordinary product. In a white noise setting, Hu and Øksendal [17] and Elliott and van der Hoek [14] define the Wick–Itô integral as a Pettis-type integral involving the derivative of fractional Brownian motion as a Hida distribution valued function. Alternatively, this integral can be interpreted as a Skorokhod integral as studied e.g. in Alòs et al. [1]. Bender [3] argues that the white-noise based definition of the Wick–Itô integral for fractional Brownian motion can be equivalently characterized by its expectations under some changes of measure, if the integral exists as an L^2-valued random variable. This S-transform approach to the Wick–Itô integral avoids most of the technicalities from white noise analysis and Malliavin calculus. In Section 2 of the present paper we extend the S-transform approach for Wick–Itô integration from fractional Brownian motion to more general Gaussian processes and explain the connection to the Wick–Riemann sum approach.

In Section 3 we introduce a transfer technique, which allows to reduce the BSDE (1) driven by the Gaussian process X to an auxiliary BSDE driven by a Brownian motion. Basically, the transfer theorems in Section 3 state that an equation of the form

$$g(X_t) = \int_0^t a(s, X_s)d^\diamond X_s + \int_0^t b(s, X_s)dV(s)$$

with deterministic functions a and b holds (including the existence of the Wick–Itô integral with respect to X), if and only if the equation

$$g(\tilde{W}_{V(t)}) = \int_0^{V(t)} a(V^{-1}(s), \tilde{W}_s)d\tilde{W}_s + \int_0^{V(t)} b(V^{-1}(s), \tilde{W}_s)ds$$

holds for a Brownian motion $\tilde{W}$, which may live on a different probability space. Here, V^{-1} is the inverse function of V and the stochastic integral is a classical Itô integral. We note that, in view of the classical Itô formula for a Brownian motion this transfer theorem immediately implies the Itô formula in the Wick–Itô sense for X, which was previously derived by various authors and by many different techniques, see Remark 3.4.

In Section 4 we apply the transfer technique to BSDEs of the form (1). We first derive generic existence and uniqueness results in terms of existence and uniqueness for an auxiliary BSDE driven by a Brownian motion. These results are then applied to the case of a Lipschitz continuous

Please cite this article in press as: C. Bender, Backward SDEs driven by Gaussian processes, Stochastic Processes and their Applications (2014), http://dx.doi.org/10.1016/j.spa.2014.03.013