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An analytical model of view factors for radiation heat transfer in planar and tubular solid oxide fuel cells

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ABSTRACT

Radiant heat transfer plays an important role in the distribution of cell temperature and current density in solid oxide fuel cells (SOFC). The objective of this paper is to introduce a mathematical model of view factors for radiation heat exchange in an in-house longitudinally distributed SOFC model. A differential view factor model is first developed for planar and tubular SOFC configurations, but is found invalid when the infinitesimal element size is comparable to the characteristic size. Then, a finite-difference view factor model is developed to solve the problem of discontinuities in the differential view factor model. Starting from a classical problem of convective and radiant heat transfer for a transparent gas flow in a gray-wall tube, a fast and accurate computation is available for the finite-difference view factor model without extra mathematical derivations of the governing equations. Compared to the simple modeling which only takes into account the surface-to-surface radiation exchange between two directly opposed elements, the detailed radiation model based on analytical view factors predicts more uniform distribution of cell temperature and current density in the overall SOFC modeling.

1. Introduction

Solid oxide fuel cells (SOFCs) are a type of high-temperature fuel cells. Planar and tubular configurations are the two most common SOFC designs. With the advantages of short current path and lower component fabrication cost, the planar design is usually the preferred design, but it suffers from sealing problems caused by thermal expansion at high-temperature. Until now, planar design has been widely used in anode-supported intermediate temperature SOFCs [1]. The tubular design is superior with respect to sealing, but often has to operate in the high temperature range of 900–1000 °C to maintain acceptable ohmic polarization due to long current paths. Some other advanced configurations, such as the Siemens Westinghouse design of flattened tubular SOFC and the integrated planar SOFC pioneered by Rolls–Royce (which combines the advantages of the planar and tubular designs by introducing the multi-cell membrane electrode assembly (MEA) concept [2]) can be considered as a variation of the two basic planar and tubular designs [1].

Radiation heat transfer plays an important role in the distribution of cell temperature and current density in SOFCs. A SOFC model with detailed radiation model is necessary for accurate prediction of cell performance. A spectral radiative heat transfer analysis within a computational fluid dynamics (CFD) SOFC model is powerful but usually computationally expensive [3]. A simple method of modeling the surface-to-surface radiation exchange is generally acceptable in thermal models of SOFCs. However, most of state-of-the-art radiation models just considered the radiation exchange between two directly opposed differential elements, where the view factor between the two finite elements is considered identical to that between two infinite diffuse interchange surfaces [4–6]. Some other papers includes radiation exchange among three adjacent control volumes, but the constant view factors are strongly dependent on the number of discrete grids [7,8]. Aiming at system-level analysis, an in-house multi-level simulation platform for solid oxide fuel cell and gas turbine hybrid generation system was recently developed in gPROMS, a commercial advanced process modeling software [9]. The objective of this paper is to introduce an analytical model of view factors in our longitudinally distributed models of planar and tubular SOFCs [10].

Fig. 1 shows the schematic diagram of a section of planar and tubular SOFCs. For planar SOFCs (PSOFC), W_{ch} and D_{ch} are the width and depth of the rectangular flow channel enclosed between the positive electrolyte negative (PEN) and the interconnect (CON). The geometry of tubular SOFCs (TSOFC) is that of the commercial West-

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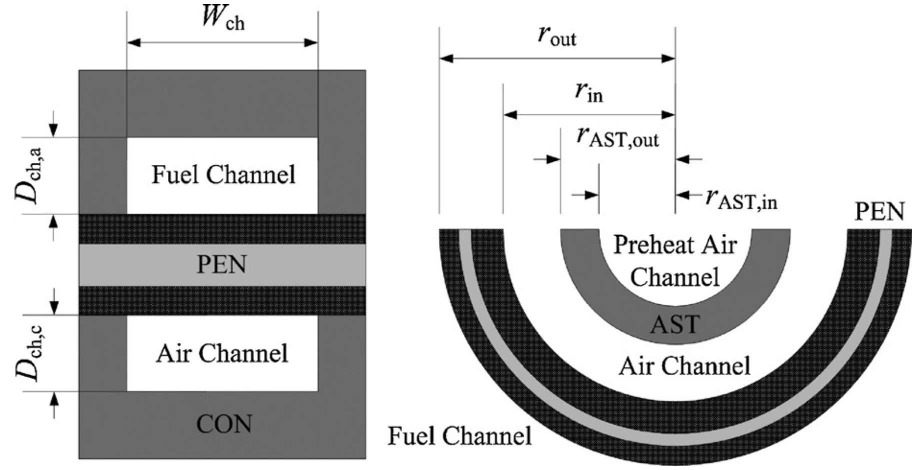

Fig. 1. Schematic diagram of a section of planar and tubular SOFCs.

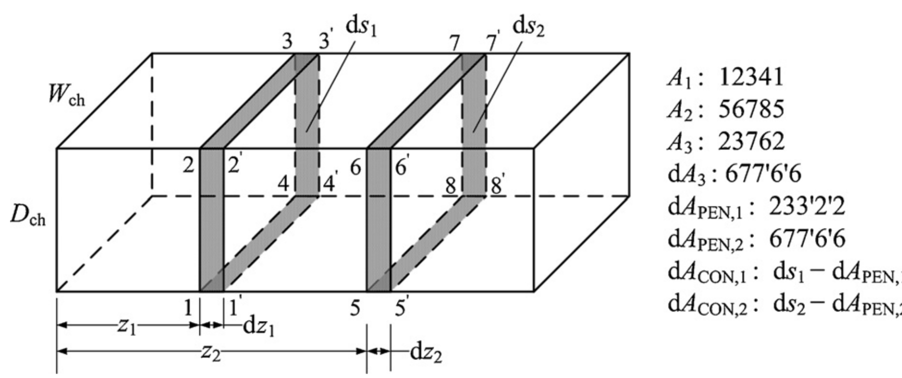

Fig. 2. Configuration for diffuse interchange in planar SOFC.

inghouse cell [1]. Fuel flows along outside of the tube, while air is first preheated via the air supply tube (AST) and then flows back into the annular area between the cathode and the air supply tube. In TSOFC, $r_{AST,in}$ and $r_{AST,out}$ are the inner and outer radius of the AST, r_{in} and r_{out} are the inner and outer radius of the cell tube. The analytical view factors between the solid phases, i.e. PEN and CON in PSOFCs, and PEN and AST in TSOFCs will be discussed in the next section.

2. Analytical differential view factors

Because the cell length (L) is generally much larger than the size in other directions, only the temperature distribution along the axial coordinate $z\in[0,L]$ is considered in our longitudinally distributed models of both planar and tubular SOFCs.

2.1. PSOFC configuration

The configuration for diffusive interchange in PSOFC is shown in Fig. 2. The view factor between two identical, parallel, and directly opposed finite rectangles ($F_{1,2}$), and the view factor between two finite rectangles of the same length, having one common edge, and 90° from each other ($F_{1,3}$), can be easily found in the view factor catalogue [11]. For the non-concave PEN surface, $dF_{dPEN1,dPEN2}=0$. According to the reciprocity and additivity rule, the view factor between two infinitesimal elements can be directly calculated from the second-order differentiation of the finite view factors, $F_{1,2}$ and $F_{1,3}$, as shown below.

Define $X=D_{ch}/W_{ch}$, $Z=z/W_{ch}$, $Y=|Z_2-Z_1|$,

$$dF_{dPEN1,\,dCON2}=dF_{dPEN1,\,ds2}=D_{ch}\frac{\partial^2F_{1,3}}{\partial z_1\partial z_2}dz_2=-\frac{1}{\pi}f_1(X,Y)dZ_2 \quad (1)$$

$$dF_{dCON1,\,dPEN2}=\frac{dA_{PEN,2}}{dA_{CON,1}}dF_{dPEN2,\,dCON1}=-\frac{1}{\pi(1+2X)}f_1(X,Y)dZ_2 \quad (2)$$

$$dF_{dCON1,dCON2}=-\frac{W_{ch}D_{ch}}{W_{ch}+2D_{ch}}\left(\frac{\partial^2F_{1,2}}{\partial z_1\partial z_2}+2\frac{\partial^2F_{1,3}}{\partial z_1\partial z_2}\right)dz_2 = \frac{2f_2(X,Y)}{\pi(1+2X)}dZ_2 \quad (3)$$

where

$$f_1(X,Y)=\frac{1}{2}\ln\left[\frac{Y^2(1+X^2+Y^2)}{(1+Y^2)(X^2+Y^2)}\right]-\frac{X^2}{(X^2+Y^2)^{3/2}}\tan^{-1}\left(\frac{1}{\sqrt{X^2+Y^2}}\right) \quad (4)$$

$$f_2(X,Y)=\frac{1}{2}\ln\left[\frac{(1+Y^2)(X^2+Y^2)}{(1+X^2+Y^2)Y^2}\right]+\frac{X}{(1+Y^2)^{3/2}}\tan^{-1}\left(\frac{X}{\sqrt{1+Y^2}}\right) \quad (5)$$