Journal of Power Sources 196 (2011) 8549–8552

Contents lists available at ScienceDirect

Journal of Power Sources

journal homepage: www.elsevier.com/locate/jpowsour

Short communication

On controllability and system constraints of the linear models of proton exchange membrane and solid oxide fuel cells

Verica Radisavljevic*

California State University Los Angeles, Department of Mechanical Engineering, CREST Center for Energy and Sustainability 5151 State University Drive, Los Angeles, CA 90032-8153, United States

ARTICLE INFO

Article history:
Received 11 April 2011
Received in revised form 4 June 2011
Accepted 7 June 2011
Available online 15 June 2011

Keywords:
Proton exchange membrane fuel cell (PEMFC)
Solid oxide fuel cells (SOFC)
Dynamic models
System integration
Controllability
System analysis

ABSTRACT

In this paper we first show that the linear models of proton exchange membrane (polymer electrolyte membrane, PEM) and solid oxide (SO) fuel cells, commonly used in power and energy literature, are not controllable. The source of uncontrollability is the equation for pressure of the water vapor that is only affected by the fuel cell current, which in fact is a disturbance in this system and cannot be controlled by the given model inputs: inlet molar flow rates of hydrogen and oxygen. Being uncontrollable these models are not good candidates for studying control of dynamic processes in PEM and SO fuel cells. However, due to their simplicity, they can be used in hybrid configurations with other energy producing devices such as photovoltaic (solar) cells, wind turbine, micro gas turbine, battery (ultra capacitor) to demonstrate some other phenomena, but not for control purposes unless the hybrid models formed in such hybrid configurations are controllable. Testing controllability of such hybrid models is mandatory. Secondly, we introduce some algebraic constraints that follow from the model dynamics and the Nernst open-loop fuel cell voltage formula. These constraints must be satisfied in simulation of considered fuel cell modes, for example, via MATLAB/Simulink or any other computer software package.

1. Introduction

The controllability and observability concepts are the system state space concepts. They have been known to control engineers for more than fifty years since the initial work of Kalman [1]. Slowly these concepts are becoming known and used in other engineering and scientific disciplines, especially when the so-called Kalman system canonical decomposition was derived in [2,3]. The Kalman canonical decomposition states that only the system modes that are both controllable and observably appear in the system transfer function and those either uncontrollable or unobservable cancel out from the transfer function (system input/output description). This result has established the fact that the state space system description (via system eigenvalues) is more general than the system description via transfer function (via system poles) since the set of system eigenvalues is broader than the set of system poles (all the poles are the eigenvalues, but not all the eigenvalues are the system poles).

In the first part of the paper we show that the commonly used linear models of PEMFC and SOFC are not controllable. In the second part of the paper we introduce some algebraic constraints on these models that follow from system dynamic equations and from the steady state analysis.

The linear mathematical model, for the PEMFC dynamics of three fundamental fuel cells dynamic variables: pressures of hydrogen, oxygen, and water vapor, was derived in 2004 [4]. The model was obtained by keeping the same state equation and slightly modifying the output equation of the mathematical model derived for the SOFC dynamics in 2000 [5]. These linear mathematical models for PEMFC and SOFC have been used in many papers including some published a year ago, see for example [6–13] for PEMFC related problems and [14–16] for SOFC related problems.

The system state space model given in [4,5] was defined by

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= -\frac{RTK_{H_2}}{V_A}x_1(t) + \frac{RT}{V_A}q_{H_2}^{in}(t) - \frac{2RTK_r}{V_A}I(t) \\
&= -\frac{1}{\tau_{H_2}}x_1(t) + \frac{1}{\tau_{H_2}K_{H_2}}q_{H_2}^{in}(t) - \frac{2K_r}{\tau_{H_2}K_{H_2}}I(t) \\
\frac{dx_2(t)}{dt} &= -\frac{RT}{V_C}K_{O_2}x_2(t) + \frac{RT}{V_C}q_{O_2}^{in}(t) - \frac{RTK_r}{V_C}I(t) \\
&= -\frac{1}{\tau_{O_2}}x_2(t) + \frac{1}{\tau_{O_2}K_{O_2}}q_{O_2}^{in}(t) - \frac{K_r}{\tau_{O_2}K_{O_2}}I(t) \\
\frac{dx_3(t)}{dt} &= -\frac{RT}{V_C}K_{H_2O}x_3(t) + \frac{2RTK_r}{V_C}I(t) \\
&= -\frac{1}{\tau_{H_2O}}x_3(t) + \frac{2K_r}{\tau_{H_2O}K_{H_2O}}I(t)
\end{aligned} \tag{1}$$

* Tel.: +1 323 343 5217; fax: +1 323 343 4555.
E-mail address: vradisa@calstatela.edu

0378-7753/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.jpowsour.2011.06.034

with the state space variables representing

$$x(t) = [\, x_1(t) \quad x_2(t) \quad x_3(t) \,]^T = [\, p_{H_2}(t) \quad p_{O_2}(t) \quad p_{H_2O}(t) \,]^T \tag{2}$$

The output equation represents the measured fuel cell voltage and it is obtained using the Nernst formula for the open-loop cell voltage, $V_0(t)$, and subtracting losses due to the cell activation, $V_{act}(t)$, and due to the stack (fuel cell) resistance, $V_{ohm}(t)$

$$\begin{aligned} y_{PEM}(t) &= V_{PEM}(t) = V_0(t) - V_{act}(t) - V_{ohm}(t) \\ &= N\left(E_0 + \frac{RT}{2F}\ln\left\{\frac{x_1(t)(x_2(t))^{0.5}}{x_3(t)}\right\}\right) \\ &\quad -B\ln(CI(t)) - R^{\text{int}}I(t) \end{aligned} \tag{3}$$

The system inputs are molar flow rates of hydrogen and oxygen, that is, $q_{H_2}(t)$ and $q_{O_2}(t)$ that can be regulated (controlled). The stack current $I(t)$ plays a role of a disturbance. Note that $CI(t)$ must be greater than 1, otherwise the activation voltage will be negative, and hence it will increase the open-loop voltage (instead of reducing it). All other coefficients are assumed to be constant. The values of the constant coefficients defined in the model equations can be found in [4].

The SOFC fuel cell model of [5] has exactly the same state Eqs. (1) and (2), but different output equation in which the activation voltage is not present in the expression for the cell output voltage, that is

$$y_{SO}(t) = V_{SO}(t) = N\left(E_0 + \frac{RT}{2F}\ln\left\{\frac{x_1(t)(x_2(t))^{0.5}}{x_3(t)}\right\}\right) - R^{\text{int}}I(t) \tag{4}$$

Of course, in the SOFC mathematical model (1), (2) and (4) the parameters take different values (except for the universal gas constant R and the Faraday constant F). The values of the constant parameters for the SOFC model defined by (1) and (4) can be found in [5].

2. Controllability of linear PEMFC and SOFC models

The importance of controllability in the design of linear controllers for PEM fuel cells was nicely demonstrated in [17], where even for originally controllable operating points of a linearized system some design techniques provide high controllability measures (requiring less control efforts and more efficient control) than the other also controllable operation points. Controllability analysis of liquid water in a fuel cell has been considered in a very recent paper [18], where it has been concluded that liquid water controllability is needed to prevent the fuel cell flooding. In this brief note we will show that the models of [4,5] are uncontrollable (zero controllability measure), meaning that no control efforts exist to satisfy general goals of transferring state variable from a given initial state to a desired final state in a finite time interval [3].

The state space model (1) can be represented in the state space form as

$$\begin{aligned} \frac{dx(t)}{dt} &= \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_{H_2}} & 0 & 0 \\ 0 & -\frac{1}{\tau_{O_2}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{1}{\tau_{H_2}K_{H_2}} & 0 \\ 0 & \frac{1}{\tau_{O_2}K_{O_2}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_{H_2}^{in}(t) \\ q_{O_2}^{in}(t) \end{bmatrix} + \begin{bmatrix} -\frac{2K_r}{\tau_{H_2}K_{H_2}} \\ -\frac{K_r}{\tau_{O_2}K_{O_2}} \\ \frac{2K_r}{\tau_{H_2O}K_{H_2O}} \end{bmatrix} I(t) \\ &= Ax(t) + Bu(t) + Gd(t) \end{aligned} \tag{5}$$

where $u(t) = [\, q_{H_2}^{in}(t) \quad q_{O_2}^{in}(t) \,]^T$ are the control inputs and $d(t) = I(t)$ denotes the system disturbance. Using the standard controllability test [3], we can form the controllability matrix for the state space system defined in (5) given by

$$\begin{aligned} C(A,B) &= \begin{bmatrix} B & AB & A^2B \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{\tau_{H_2}K_{H_2}} & 0 & -\frac{1}{\tau_{H_2}^2K_{H_2}} & 0 & \frac{1}{\tau_{H_2}^3K_{H_2}} & 0 \\ 0 & \frac{1}{\tau_{O_2}K_{O_2}} & 0 & -\frac{1}{\tau_{O_2}^2K_{O_2}} & 0 & \frac{1}{\tau_{O_2}^3K_{O_2}} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{6}$$

It is obvious that the rank of the controllability matrix $C(A, B)$ is equal to 2, that is

$$\text{rank}\{C(A,B)\} = 2 < 3 = n \tag{7}$$

This indicates that in the third-order dimensional linear system considered only two state variables are controllable, and the third one is uncontrollable. Examining the state space equations, it can be observed that the equation for the water vapor is not affected by the control input signal and hence, the water vapor pressure is the uncontrollable variable in this system.

The system controllability in this particular model is needed for several reasons. First of all, it is well known that being uncontrollable, the state variable $x_3(t)$ will not appear in the system transfer function [3], which in this case means that the system transfer function is of order two, corresponding to the controllable state variables $x_1(t)$ and $x_2(t)$. Hence, every frequency domain analysis that involves model (1) will be superficial since it will not involve the state variable $x_3(t)$. Secondly, it is known from [3] that state feedback can be used to stabilize unstable systems, but it can not make uncontrollable systems controllable so that the variable $x_3(t)$ by no means can be affected by control input signals, and it will remain affected only by the disturbance signal $I(t)$ that changes randomly as $I(t) = V_{fc}(t)/R_L$, where the load R_L changes randomly in time as a piecewise constant. Hence, changes in the dynamics of the state variable $x_3(t)$ will be fully determined only by its time constant and the fuel cell disturbance (current). It should be emphasized that according to the numerical data from [4] and [5] the time constant for $x_3(t)$ is much larger than for the remaining two state variables ($\tau_{H_2O} = 18.418\,\text{s}$, $\tau_{O_2} = 6.64\,\text{s}$, $\tau_{H_2} = 3.37\,\text{s}$ for PEMFC [4], and $\tau_{H_2O} = 78.3\,\text{s}$, $\tau_{O_2} = 2.91\,\text{s}$, $\tau_{H_2} = 26.1\,\text{s}$ for SOFC [5]) which means that $x_3(t)$ takes much longer time to reach its steady state value (when it will be dictated only by the steady state value of the current $x_3^{ss} = 2K_rI^{ss}/K_{H_2O}$) than the remaining two state variables. Moreover, the magnitude of the state variable $x_3(t)$ is much smaller than the magnitude of the state variables $x_1(t)$ and $x_2(t)$, and since it appears in the dominator of the cell output voltage formula (4), it will have a more dominant, more lasting, and more unpredictable impact on the cell output voltage. Thirdly, controlling water in a fuel cell is fundamentally important [18], since it can cause cell flooding, degrade the cell polarization curve, and eventually damage the cell membrane [19,20] (note that the water vapor mass m_{H_2O} is proportional to the water vapor pressure p_{H_2O} [19]).

3. System analysis constraints of PEMFC and SOFC models

In this section, we derive some algebraic constraints that follow from the model differential equations. These constraints were not imposed in the papers [4,5] that derived the considered models nor in any other follow-up paper that have used these models alone or in hybrid configurations with other electric energy generating devices. The constraints are imposed at steady state, for the initial conditions, and for all time instants.