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Distribution of steel fibres in rectangular sections

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Abstract

In this paper a calculation method is explained to predict the total number of fibres crossing a rectangular section. The largest part of the paper deals with the theoretical calculation of an orientation factor. The orientation factor is defined here as the average length of the projection on the longitudinal axis of all fibres crossing a section, divided by the fibre length. Once the orientation factor is found, a simple calculation gives the number of fibres crossing a crack. Since the proposed approach is to a large extent new, there is a need for verification with test results. For this reason a fibre counting was done on 107 Rilem beam specimens, involving different fibre types. The comparison with the calculated number of fibres shows that the model provides good predictions of the number of fibres crossing a section.

Keywords: Fibre distribution; Steel fibre concrete; Fibre orientation

1. Introduction

One of the most important properties of steel fibre concrete is its ability to transfer stresses across a cracked section. This ability is mostly translated into a toughness parameter [1], which is a measure for the energy consumed during a bending test. Experimental research at the Department of Civil Engineering of the K.U. Leuven and also elsewhere [2] has shown that there is a high degree of proportionality between the toughness and the number of effective fibres that are counted in the cracked section. A fibre was considered to be effective if the hook of the fibre was straightened after the two beam halves were separated. This conclusion creates an interest in knowing the number of effective fibres that cross a cracked section. The number of effective fibres is not only dependent on the fibre dosage, but also on the orientation factor and the length efficiency factor [3,4]. In this paper only the total number of fibres (effective as well as non-effective) is determined. For further calculations it could be assumed that the number of effective fibres is proportional to the number of total fibres. The calculation of this coefficient of proportionality, which is dependent on the efficiency of the fibre, is not considered in this paper.

2. General approach

To calculate the total number of fibres, it is essential to know the orientation factor. It has been shown by Krenchel [5] that the number of fibres can be found as follows:

$$n = \alpha \frac{V_f}{A_f}$$

where n = number of fibres per unit surface; α = orientation coefficient; V_f = fibre volume fraction; A_f = cross-section of a fibre.

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The calculation of the orientation factor α has been the interest of many researchers [2–10]. First the orientation factor is calculated for the case the fibre can rotate freely in all directions. This is the case for a fibre in bulk (zone 1 in Fig. 1). Secondly one boundary condition is considered, parallel to the direction in which the orientation factor is determined. This simulates the approximity of one mould side (zone 2 in Fig. 1). And finally a second boundary condition is added, also parallel to the direction in which the orientation factor is determined, but now perpendicular to the first boundary condition. This simulates a fibre situated in a corner of the mould (zone 3 in Fig. 1).

b and h (see Fig. 1) are the width and the height of the beam section, while l_f stands for the fibre length. The following seven assumptions are made for calculating the orientation factor in each of these three areas:

1) The fibres are straight. For hooked end fibres the same orientation factor can be taken since the effect of the hooks is negligible on the orientation factor.
2) If the fresh concrete is vibrated for a long time or when it has a high workability (e.g. self compacting concrete), the fibres tend to orient in a horizontal plane [11]. This orientation effect depends highly on the vibration time and frequency and the workability and composition of the concrete and it is therefore very difficult to quantify. However, from other research [12] it is concluded that the vibration does not have a significant effect on the orientation if the specimen is only vibrated for 1 or 2 min and if the workability of the fresh concrete is not too high. The effect of vibration on the orientation of the fibres is not considered here.
3) The location of the fibre in the beam is characterised by its point of gravity. Each point of the cross-section is considered to have an equal probability of being the gravity point of a fibre. Some researchers have found that there is an under reinforcement in the zones near to the boundaries while others have found the opposite. Here no preference is made [2,7].
4) The fibre orientation in area 1 (Fig. 1) is not influenced at all by the boundary conditions.
5) The fibre orientation in area 2 (Fig. 1) is only influenced by one side of the mould.
6) The fibre orientation in area 3 (Fig. 1) is influenced by two sides of the mould.
7) The top surface of the section is assumed to have the same boundary condition as the sides of the mould. After casting, this surface is smoothened so that there are no fibres sticking out. There could be a higher number of fibres at the surface due to the topping off and levelling of the specimen. This effect is not considered in this paper.

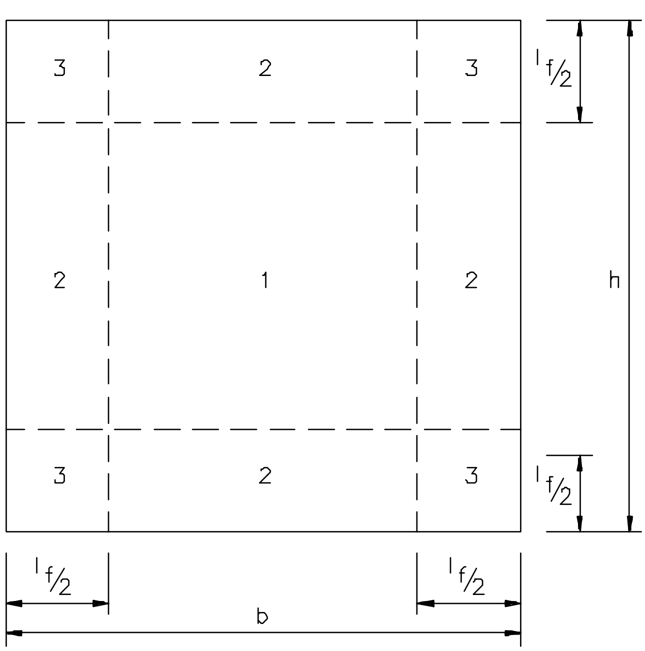

Fig. 1. Cross-section of a beam divided into three different orientation zones.

When the orientation factor for the areas 1, 2 and 3 (Fig. 1) are known to be α_1, α_2 and α_3, respectively, then the overall orientation factor can be calculated as follows by taking the geometrical average over the section:

$$\alpha = \frac{\lfloor \alpha_1 \times (b - l_f)(h - l_f) + \alpha_2 \times [(b - l_f)l_f + (h - l_f)l_f] + \alpha_3 \times l_f^2 \rfloor}{bh} \tag{1}$$

with α_1 = orientation factor in zone 1 (Fig. 1); α_2 = orientation factor in zone 2 (Fig. 1); α_3 = orientation factor in zone 3 (Fig. 1).

3. Orientation factor in bulk

A fibre in bulk (zone 1 in Fig. 1) is not limited by any boundary condition and can rotate freely round its gravity point. If all the possible orientations of the fibre are considered, the end points of the fibre describe the surface of a sphere. Each point on the sphere has an equal probability to be the end of the fibre. This means that the probability that the fibre makes an angle θ with the longitudinal axis of the beam is proportional to the area dA (Fig. 2) with:

$$\mathrm{d}A = \underbrace{\frac{\pi l_f^2}{2}}_{A1} \sin\theta \,\mathrm{d}\theta \tag{2}$$

The contribution of the area dA to the orientation factor is then $\cos\theta\,\mathrm{d}A$. Integrating this over half the sphere and dividing by the surface of half the sphere gives: