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A simplified isotropic damage model for concrete under bi-axial stress states

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Abstract

This paper presents a concrete model that is capable of describing the response of concrete under bi-axial loading, with the features of simplicity and avoidance of convergence problems, often seen in plasticity based models. The proposed model incorporates the failure of concrete into a conventional continuum damage mechanics framework, where particular emphasises are placed on highlighting the different responses of concrete under tension and compression, as well as the different contributions of hydrostatic and deviatoric stress components on concrete damage. A weighted damage parameter and a damage multiplier are introduced to eliminate potential convergence problems and to reduce the effect of hydrostatic pressure on damage, respectively. Finally, several examples are provided and compared with experimental data.

Keywords: Concrete model; Isotropic damage; Bi-axial loading; Finite element analysis

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1. Introduction

The mechanical response of concrete is weakened by the development of micro-cracks and is mainly characterised by strain softening, progressive deterioration, volumetric dilatancy, and induced anisotropy. From plasticity, damage and fracture mechanics viewpoints, these phenomena can be considered as a combination of unrecoverable plastic deformation, the degradation of material stiffness and the nucleation, growth and interaction of those defects, presented in Fig. 1 for uniaxial loading. In order to model the complicated responses of concrete-like materials, different mechanics theories can be adopted. In general, to determine plastic deformation classical plasticity theory is effective; to describe strain-softening, one can use either plasticity theory or continuum damage theory; to simulate crack opening and closure, fracture mechanics, plasticity and contact methods can be employed. However, the simplest and most effective method to model stiffness degradation is through continuum damage mechanics, which assumes cracking reduces the Young's modulus of the material.

Various damage mechanics based methods for modelling the response of concrete have been developed and various mechanisms for describing its behaviour have been suggested. Amongst these are plastic-damage models [1–4], coupled elastoplastic-damage models [5–11], and friction-damage models [12–16], to mention just a few. In the plastic-damage approach, the thermodynamic potential is decomposed into two parts. One is related to elastic behaviour and the other to plastic response. The latter may be modelled by either plasticity theory or by directly introducing a plastic term that is associated with a damage internal variable, such as in the work of La Borderie et al. [4]. In the coupled

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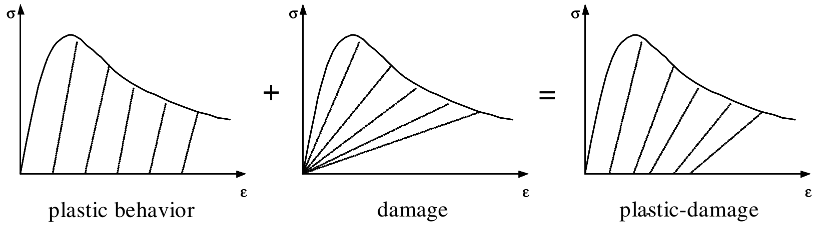

Fig. 1. Plastic-damage behaviour of concrete materials.

elastoplastic-damage model, the concept of stiffness degradation is coupled with classical plasticity theory. As for friction-damage models, micro-crack growth is coupled with friction related to dissipative mechanisms. The friction sliding over internal crack surfaces is assumed to have a plasticity kind of behaviour. However, among these, a proper description of strain softening and stiffness degradation by means of damage mechanics is essential.

The application of continuum damage mechanics theory to concrete dates from the late 1970s. Typically, these include the scalar damage models [17], unilateral damage models [12,18], models with permanent strains, induced anisotropy or high compressive stresses [19,13], models which include anisotropic damage and dilation [3,20], as well as thermo-mechanical damage [21,22]. Among the various scalar damage models, Mazar's is perhaps more popular due to its simplicity.

Generally, a typical reverse cyclic loading on concrete may be described by six stages, see Fig. 2, i.e. elastic tension upon yielding (segment ab), subsequently tensile hardening/softening (segment bc), then elastic unloading (segment cd); once into compressive state, starting with compressive loading until yielding (segment de), then compressive hardening/softening (segment ef), finally reverse unloading (segment fg). Corresponding to these processes, concrete experiences changes from initiation/nucleation of micro-cracks, propagation and clustering of existing cracks, crack closure under unloading process, moduli recovery under compressive loads (unilateral effect), and partial crushing. From a damage mechanics standpoint, they can be simply described as (1) degradation of stiffness by a tensile damage parameter; (2) hardening/softening of tensile damage surface by a proper hardening/softening parameter, i.e. an evolution law related to tensile strain or stress; (3) elastic unloading with constant stiffness; (4) stiffness recovery by activating compressive damage parameter; and then (5) hardening/softening of compressive damage surface and stiffness degradation controlled by another evolution law related to compressive strain or stress; and finally (6) compressive unloading with constant stiffness.

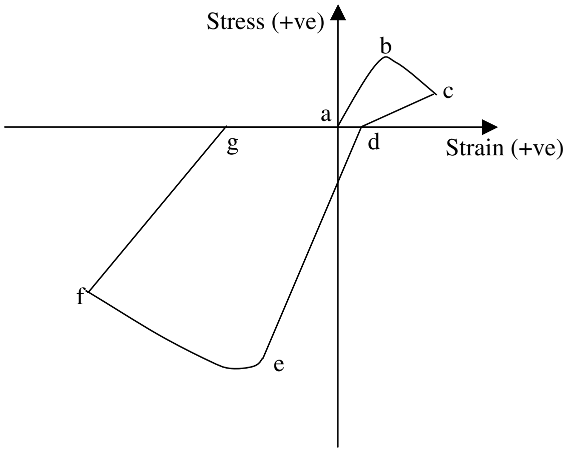

Fig. 2. Typical concrete response under reverse uniaxial tensile/compressive loading.

How to activate and inactivate tensile and compressive damage evolution laws is a key issue, which requires information on tensile/compressive stress/strain state. A common practice is to separate stress and strain into positive and negative parts in some existing models. Tensile and compressive damage are independently calculated by means of corresponding damage parameters with the change in load. This treatment is feasible and reasonable for monotonic uniaxial loading where only one of damage surfaces (tensile or compressive) dominates damage evolution. However, convergence problems would arise under bi-axial and reverse cyclic loading, or even uniaxial reverse cyclic loading due to the conflict of tensile and compressive damage surfaces induced by the Poisson's ratio effect. In a uniaxial reverse cyclic loading case, for instance, tensile damage may (quickly) exceed the major compressive damage due to transverse expansion under compression, which, in turn, may result in a wrong judgement for activating tensile or compressive evolution laws and lead to premature convergence difficulties.

The model introduced in this paper is motivated by the problem encountered in modelling pullout response of fibre reinforced concrete by classical plasticity theory, where severe spalling of cement matrix may cause convergence problem. It is required to develop a bi-axial isotropic concrete model, with the principle of being simple (e.g. a few parameters and little computing complexity) and easy to implement in existing commercial FE packages, but with less convergence problems. In the contribution, we start with a simple outline of the damage mechanics framework, and then follow by