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Influence of a finite notch root radius on fracture toughness

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Abstract

The validity of fracture toughness data from tests with V-notched bending bars depends in the notch root radius and the presence of an *R*-curve behaviour. In a theoretical study it is shown how the notch radius affects the formally computed conventional toughnesses. These are computed under the assumption that the introduced notch with a small crack at the notch root acts as a long crack of the same total size and, in a stronger simplification, that the crack length is identical with the depth of the notch.

Keywords: Crack tip toughness; Fracture toughness; Notch effect; *R*-curve; Testing

1. Introduction

Most investigations of fracture toughness deal with cracks starting from narrow notches. These are introduced in test specimens by thin saw cuts or produced with the razor blade procedure as proposed by Nishida et al.[1] and successfully applied by Kübler.[2] If a_0 is the depth of the notch and ℓ the length of an edge crack propagating from the notch root (for the geometric data see Fig. 1a), the stress intensity factor commonly, but incorrectly is computed as the stress intensity factor for a crack of total length $a = a_0 + \ell$

$$K^* = \sigma_{\text{bend}}\sqrt{\pi(a_0+\ell)}\, F_{\text{bend}}\left(\frac{a}{W}\right) \tag{1}$$

where F_{bend} is the geometric function for an edge crack of length $a = a_0+\ell$ in a specimen of width W under the applied load, here, for instance, under bending load. The geometric function is available from fracture mechanics handbooks. The formally computed "apparent stress intensity factor" K^* given by Eq. (1) is the correct value only in cases where the crack length ℓ is clearly larger than the radius of the notch. In the first crack extension phase where the crack length ℓ is comparable to R, Eq. (1) does not represent the correct stress intensity factor value.

In fracture toughness tests the stress intensity factor is often computed with the notch depth a_0 as the crack length. This is necessary in all cases where the crack length at failure cannot be identified on the fracture surface. This value may be denoted here by $\hat{K}$ with

$$\hat{K} = \sigma_{\text{bend,max}}\sqrt{\pi a_0}\, F_{\text{bend}}\left(\frac{a_0}{W}\right) \tag{2}$$

It is clear that in the presence of a strong *R*-curve behaviour with an extended stable crack growth phase before final fracture, Eq. (2) badly describes the fracture toughness, even if notch effects are negligible. It is the aim of this contribution to show the influence of the notch root radius and *R*-curve on the formally computed toughnesses according to Eqs. (1) and (2).

2. Notch effect and *R*-curve

To examine the case of a material with *R*-curve, the fracture mechanics problem of a small crack in front of a finite notch has to be considered. In the special case of an edge crack ahead of a slender notch with R being small compared to the crack length and the other specimen dimensions, the true stress intensity factor K is given by[3]

$$\frac{K}{K^*} = \tanh\left(2.243\sqrt{\frac{\ell}{R}}\right) \tag{3}$$

This relation is shown in Fig. 2 as the dashed curve. If a semi-elliptical crack is assumed (for the geometric data see

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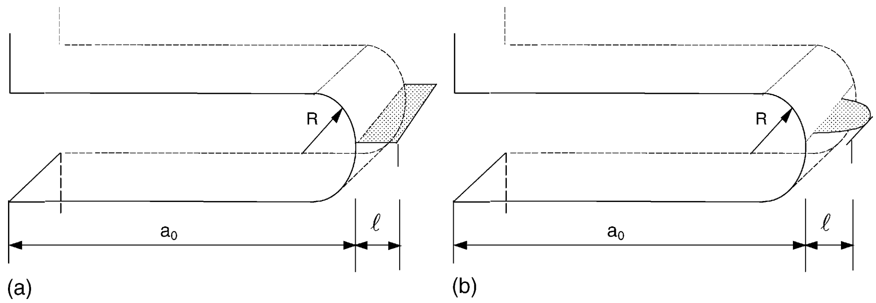

Fig. 1. Cracks in front of a narrow notch: (a) edge crack and (b) semi-elliptical crack.

Fig. 1b), then as shown in[4]

$$\frac{K}{K^*} \cong \tanh\left(2.243 g\left(\frac{\ell}{R}\right)\sqrt{\frac{\ell}{R}}\right) \quad (4)$$

with

$$g\left(\frac{\ell}{R}\right) \cong \frac{2}{3} + 0.178\left(1 - \exp\left(-1.64\frac{\ell}{R}\right)\right) \quad (5)$$

In Fig. 2 this relation is shown by the solid curve. From the curves of Fig. 2a, it is clearly visible that the true stress intensity factor is significantly lower than the formally computed values of K^*, irrespective of the special crack shape. On the other hand, it can be concluded that notch effects are without importance for $\ell > 1.5R$. For the following numerical evaluations, Eq. (3) will be used.

Replacing of K^* in Eq. (3) by this formally computed "apparent fracture toughness" yields

$$K \cong \hat{K}\sqrt{\frac{a_0+\ell}{a_0}\,\frac{F(a/W)}{F(a_0/W)}}\tanh\left(2.243\sqrt{\frac{\ell}{R}}\right) \quad (6)$$

Sometimes, this relation was successfully used for a fracture toughness evaluation,[2,5–7] where the rough approximation

$$\sqrt{\frac{a_0+\ell}{a_0}\,\frac{F(a/W)}{F(a_0/W)}} \approx 1 \Rightarrow K \approx \hat{K}\tanh\left(2.243\sqrt{\frac{\ell}{R}}\right) \quad (7)$$

was made, which is applicable for $\ell \ll a_0$ at least. Damani et al.[5,6] further assumed ℓ to be proportional to the size of defects at the notch root caused by notch preparation or to the mean grain size, since grain boundaries may act as crack-like defects. Errors in toughness determination are unavoidable, at least in the case of a significant stable crack extension before failure, as expected for materials with a pronounced R-curve behaviour. In this case, the maximum load indicated by the solid circle in the load versus displacement curve of Fig. 2b is commonly introduced in Eq. (2), although the load versus displacement plot shows a clear deviation from the initial straight line, starting at the open circle.

In a material with an R-curve effect, the externally applied stress intensity factor K_{appl} and the intrinsic shielding stress intensity factor have to be superimposed in order to obtain the total stress intensity factor K_{total}

$$K_{total} = K_{appl} + K_{sh} \quad (8)$$

which governs the crack tip stress field. Fig. 3a represents these stress intensity factor contributions.

Stable crack propagation occurs under the condition of the total stress intensity factor equalling the so-called crack tip toughness K_{I0}

$$K_{total} = K_{I0} \quad (9)$$

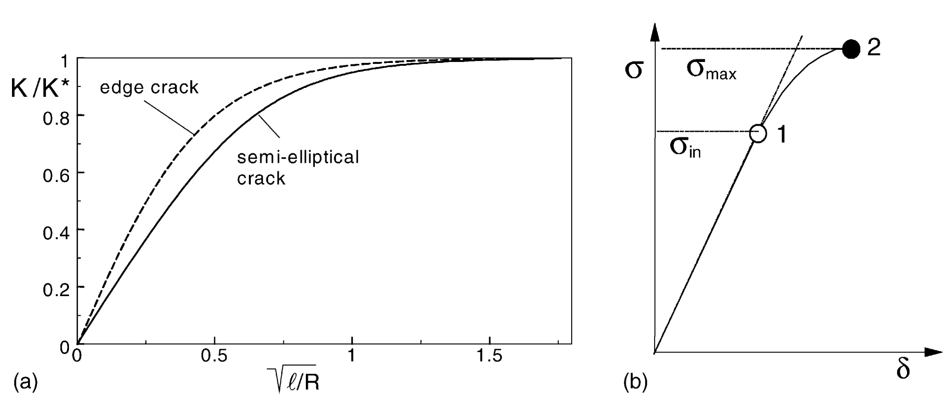

Fig. 2. (a) Ratio of true stress intensity factor K and formally computed stress intensity factor K^* as a function of ℓ/R, (b) stresses in a fracture toughness test, σ_{in} = stress at the first deviation from the initial straight line of the load vs. displacement plot, σ_{max} = stress at failure.