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Novel analysis for nanoindentation size effect using strain gradient plasticity

Hunkee Lee *, Seonghyun Ko, Junsoo Han, Hyunchul Park, Woonbong Hwang

Department of Mechanical Engineering, Pohang University of Science and Technology, San 31, Hyoja-dong, Namgu, Pohang, Gyungbuk 790-784, Republic of Korea

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Abstract

A model describing nanoindentation as plastic deformation resulting from a strain gradient is investigated. Using a simplified indentation model, the effective strain and effective strain gradient for indentation depth are derived. To validate the proposed model, solutions for depth-dependent hardness of Ag, Al, Cu and Ni are compared with experiments.

Keywords: Strain gradient plasticity; Nanoindentation; Size effect; Depth-dependent hardness

1. Introduction

Recent experiments on non-uniform plastic deformation have shown a size effect at the micro/nano scale. For example, experiments on nanoindentation showed increasing indentation depth reduces material hardness [1–5]. Similar results were also found in two other experiments. A torsional experiment with micron-diameter copper wire confirmed that reducing the wire diameter increases shear strength [6], and a thin beam bending experiment has shown that thinner beams have higher strength [7].

The size effect can also be found in fracture. In the crack tip, it has been proven that there is a difference between the stress calculated by classical mechanics (HRR field, K field) and the stress to create a crack [8].

Classical continuum plasticity cannot predict these size effects. One of the reasons is that the constitutive equation of classical mechanics does not include constituent internal length as a parameter for deformation. Several theories have been suggested in order to explain deformation at the micro/nano scale. One of these theories is strain gradient plasticity, which explains the size effect very well [9–11]. The strain gradient plasticity theory is based on Taylor's hardening model. This research proposes an analytic model for nanoindentation and also explains the indentation size effect well.

Using a simplified model for an axisymmetric indentation, the effective strain and effective strain gradient for the indentation depth are derived and the depth-dependent hardness is predicted. Hardness predicted by the proposed model is compared with experimental results. The proposed model predicts the size effect very well.

2. Strain gradient plasticity—overview

Taylor's hardening equation describing the relation between shear stress and dislocation density in the material is expressed as follows:

* Corresponding author. Tel.: +82 54 279 2837; fax: +82 54 279 5899.
E-mail address: qq9481@postech.ac.kr (H. Lee).

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$$\tau = \alpha\mu b\sqrt{\rho_T} = \alpha\mu b\sqrt{\rho_S + \rho_G}, \tag{1}$$

where τ is the shear stress, ρ_T the total dislocation density, ρ_S the density of the statistically stored dislocations (SSD), ρ_G the density of the geometrically necessary dislocations (GND), μ the shear modulus, b the Burgers vector and α an empirical constant usually ranging from 0.2 to 0.5. The gradient in the strain field is accommodated by the GND, so that the effective strain gradient η can be defined by

$$\eta = \rho_G b. \tag{2}$$

If the von Mises rule is used, the tensile flow stress is described as

$$\sigma = \sqrt{3}\tau = \sqrt{3}\alpha\mu b\sqrt{\rho_S + \rho_G}. \tag{3}$$

From the power-law hardening rule for stress and strain, we have

$$\sigma = \sigma_{\mathrm{ref}}\varepsilon^N, \tag{4}$$

where σ_{ref} is the reference stress for the uniaxial tension, ε the effective strain and N the work hardening exponent ($0 \leqslant N < 1$).

From Eqs. (1)–(4), the flow stress for the strain gradient plasticity is obtained as

$$\sigma = \sigma_{\mathrm{ref}}\sqrt{\varepsilon^{2N} + l\eta}, \tag{5}$$

where the characteristic length l is the material parameter described as follows:

$$l = 3\alpha^2\left(\frac{\mu}{\sigma_{\mathrm{ref}}}\right)^2 b. \tag{6}$$

3. Application to nanoindentation

For modeling the nanoindentation process, the indenter tip is modeled as a rigid circular cone as shown in Fig. 1, where a is the radius of the contact area and h the indentation depth. Indenter tip angle changes according to the type of the tip. In this research, the indenter tip angle is converted to 70.32° ($A = 24.56h^2 = \pi a^2$, $\tan\vartheta = h/a$), that is equivalent to the Berkovich tip.

As the tip is indented into the specimen, the GND break out for the plastic deformation at the surface. Taking into account the assumption that the contact area does not change between loading and after the load has been removed for indentation, the plastic zone is assumed to be limited to the hemispherical region with radius a.

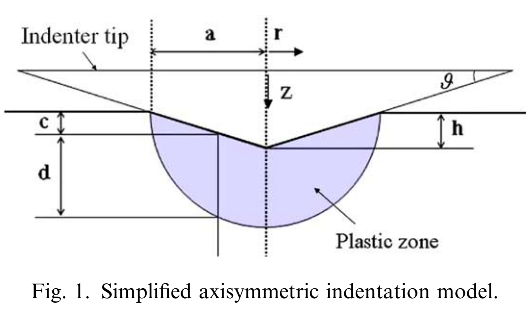

Fig. 1. Simplified axisymmetric indentation model.

From Fig. 1, the normal strain in the direction of indentation is given by

$$\varepsilon_{zz} = -\frac{c}{c+d} = -\sqrt{\frac{a-r}{a+r}}\tan\vartheta. \tag{7}$$

In the cylindrical coordinate system, assuming the material is incompressible, then

$$\frac{u_r}{\partial r} + \frac{u_r}{r} + \frac{u_z}{\partial z} = 0 \tag{8}$$

must hold. Assuming u_r is a function of r, the displacement field is obtained as follows:

$$u_z = -\left(\sqrt{\frac{a-r}{a+r}}\tan\vartheta\right)z - r\tan\vartheta + h \tag{9}$$

$$u_r = \frac{1}{r}\left\{\frac{1}{2}(r-2a)\sqrt{a^2-r^2} - a^2\sin^{-1}\sqrt{\frac{a+r}{2a}} + a^2\left(1+\frac{\pi}{4}\right)\right\}\tan\vartheta. \tag{10}$$

The strain components are determined to be

$$\varepsilon_{rr} = \left\{\frac{1}{2}(2a-r)\frac{\sqrt{a^2-r^2}}{r^2} + \sqrt{\frac{a-r}{a+r}} + \frac{a^2}{r^2}\sin^{-1}\sqrt{\frac{a+r}{2a}} - \frac{a^2}{r^2}\left(1+\frac{\pi}{4}\right)\right\}\tan\vartheta \tag{11}$$

$$\varepsilon_{\vartheta\vartheta} = \frac{1}{r^2}\left\{\frac{1}{2}(r-2a)\sqrt{a^2-r^2} - a^2\sin^{-1}\sqrt{\frac{a+r}{2a}} + a^2\left(1+\frac{\pi}{4}\right)\right\}\tan\vartheta \tag{12}$$

$$\varepsilon_{rz} = \frac{1}{2}\left\{\frac{az}{(a+r)\sqrt{a^2-r^2}} - 1\right\}\tan\vartheta. \tag{13}$$

The z component in Eqs. (9) and (13) is obtained from the fact that total volume does not change in the plastic zone $\left(\frac{1}{3}\pi a^2 h \times \frac{1}{3}h = \left(\frac{2}{3}\pi a^3 - \frac{1}{3}\pi a^2 h\right)z\right)$, therefore

$$z = \frac{a\tan^2\vartheta}{3(2-\tan\vartheta)}. \tag{14}$$

Then the effective strain is obtained by Eq. (14)

$$\varepsilon = \sqrt{\frac{2}{3}\left(\varepsilon_{rr}^2 + \varepsilon_{\vartheta\vartheta}^2 + \varepsilon_{rz}^2 + \varepsilon_{zz}^2\right)}. \tag{15}$$

The effective strain from the proposed model and Zhao et al.'s model for an indentation depth of 2 μm is shown in Fig. 2. Zhao et al. [12] considered the indentation process to be pure compression, so shear strain does not occur during indentation.