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Orbital angular momentum crosstalk of single photons propagation in a slant non-Kolmogorov turbulence channel

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ABSTRACT

We analyze the orbital angular momentum (OAM) crosstalk of single photons propagation through low-order atmospheric turbulence. The probability models of the orbital angular momentum crosstalk for single photons propagation in the channel with the non-Kolmogorov turbulence tilt, coma, and astigmatism and defocus aberration have been established. It is found, for $\alpha=11/3$, that the turbulent tilt is the dominant aberration which causes the orbital angular momentum crosstalk, the coma is second and the astigmatism is third, but the defocus aberration has no impact on OAM. The results also indicate that the regularities of orbital angular momentum crosstalk caused by the tilt, the coma and the astigmatism are almost the same, respectively. The crosstalk probability of the orbital angular momentum increases as the azimuth mode index p of Laguerre–Gaussian (LG) beam increases, the turbulent strength C_n^2 enhances, the orbital angular momentum quantum number rises, the diameter of circular sampling aperture D and the channel zenith angle θ increase.

1. Introduction

Several models have recently been established to use the orbital angular momentum (OAM) states of light as a basis set for impressing quantum information onto single-photon light field propagation in turbulence atmosphere [1–4]. A key motivation for this idea is that the OAM states provide an infinite orthonormal basis set for describing the transverse structure of the beam [5]. However, the spatial-structure nature of OAM implies that it may be susceptible to atmospheric turbulence [3]. Many investigation results for the effects of atmospheric turbulence on the OAM states of photons propagation in atmosphere optical communication channel have been reported, such as, C. Paterson [3,4] investigated the effect of Kolmogorov atmospheric turbulence aberrations on free-space optical communication using angular momentum states of single photons with the pure phase perturbation approximation of turbulent aberrations. J.A. Anguita et al. [6] numerically analyzed the effects of atmospheric turbulence on the multichannel free-space optical communication system based on OAM-carrying beams and find that turbulence induces attenuation and crosstalk among channels. A.T. Glenn et al. [1] analyzed the influence of atmospheric turbulence on propagation of quantum states of light carrying orbital angular momentum and given a result that quantifies the rate at which quantum information encoded the OAM states of individual photons is lost as a result of propagation through atmospheric turbulence. Zhang et al. [2] modeled the effects of atmospheric turbulence tilt, defocus, astigmatism and coma aberrations on the orbital angular momentum measurement probability of photons propagating in weak turbulent regime.

In the current paper, we model the crosstalk probability of the orbital angular momentum (OAM) for photons propagation through the low-order non-Kolmogorov atmospheric turbulence based on the Zernike polynomial expansion of turbulence-induced phase aberrations.

In Section 2 we model the crosstalk probability of the orbital angular momentum states for single photons propagation through the tilt, coma, astigmatism and defocus aberration non-Kolmogorov turbulent channel. In Section 3 we analyze the effects of the tilt, coma, astigmatism and defocus aberration on orbital angular momentum (OAM) crosstalk by numerical simulations and conclusions are given in Section 4.

2. Crosstalk probabilities of orbital angular momentum states

The modes $LG_{l_0,p}(r,\varphi,z)$ of Gauss–Laguerre (LG) beam propagating in a free-space have the eigenfunction of orbital angular momentum operator $\hat{L}_z=-i\hbar\partial/\partial\varphi$ [7]. Therefore, we make use of the LG beam modes to study the Crosstalk effects of the orbital angular momentum for the transmitted photons in a turbulent channel. Using cylindrical polar coordinates (r,φ,z) and defining the z-axis as the propagation direction along the intensity centre of the beam. The normalized $LG_{l_0,p}$ model at z is given by [7]

$$LG_{l_0,p}(r\varphi z)=R_{l_0,p}(r,z)\frac{\exp(il_0\varphi)}{\sqrt{2\pi}}\exp[-i(2p+|l_0|+1)\delta] \quad (1)$$

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where the parameters l_0 and p are the radial and azimuthal mode indices respectively. l_0 is also the orbital angular momentum quantum number for photon in the mode. φ is the azimuthal angle, r is the radial cylindrical coordinate and δ is the Gouy phase. The radial orthonormal basis functions $R_{l_0,p}(r)$ of the field distribution of the LG beam model with eigenvalues of orbital angular momentum $l_z = l_0\hbar$ are of the form [2]

$$R_{l_0,p}(r,z) = \frac{1}{w}\sqrt{\frac{2p!}{(p+|l_0|)!}}\left(\frac{r}{w}\right)^{|l_0|} L_p^{l_0}\left(\frac{r^2}{w^2}\right)\exp\left(-\frac{r^2}{2w^2}\right)\exp\left(-\frac{ikr^2}{4R}\right) \tag{2}$$

where $L_p^{l_0}(\cdot)$ is the generalized Laguerre polynomials. $w = w_0\sqrt{1+(z/z_R)^2}$ is the spot size of bean, $z_R = \frac{1}{2}kw_0^2$ is Rayleigh distance, w_0 is beam waist and $R = z[1+(z_R/z)^2]$ is phase front radius of curvature.

Using the Rytov approximation [8], the LG model which is propagation in the weak turbulence regime and at z can be represented as

$$LG_{l,p}(r,\varphi,z) = LG_{l_0,p}(r,\varphi,z)\exp[iS(r,\varphi,z)] \tag{3}$$

here $S(r,\varphi,z) = a_1 + S_{tilt} + S_{defo} + S_{asti} + S_{coma}\cdots$ being the total complex phase perturbation of the field due to random inhomogeneous along the propagation channel [9,10], a_1 being the constant phase, $S_{tilt}(r,\varphi) = 2a_2 r\cos\varphi + 2a_3 r\sin\varphi$ being the turbulence Z-tilt aberration, $S_{defo}(r,\varphi) = a_4\sqrt{3}(2r^2-1)$ being the turbulence defocus aberration, $S_{asti}(r,\varphi) = \left[a_5\sqrt{6}r^2\sin2\varphi + a_6\sqrt{6}r^2\cos2\varphi\right]$ being the turbulence astigmatism aberration, $S_{coma}(r,\varphi) = a_7\sqrt{8}(3r^3-2r)\sin\varphi + a_8\sqrt{8}(3r^3-2r)\cos\varphi$ being turbulence coma aberration, a_i being the expansion coefficient of the Zernike polynomials [11,12] and D being the diameter of circular sampling aperture.

As the beam propagates through the atmosphere, the effect of the refractive index fluctuations perturbs the complex amplitude of the wave so that it is no longer guaranteed to be in the original eigenstate of orbital angular momentum. The resulting wave now can be written as a superposition of eigenstates [7].

$$LG(r,\varphi,z) = \sum_p\sum_l a_{l,p}(z)LG_{l,p}(r,\varphi,z) \tag{4}$$

where $a_{l,p}(z)$ is the expansion coefficient,

$$a_{l,p}(z) = \iint R_{l,p}^*(\rho)\frac{\exp(-il\varphi)}{\sqrt{2\pi}}\exp[i(2p+|l|+1)\delta]LG(\rho,\varphi,z)\rho d\rho d\varphi \tag{5}$$

here * denotes complex conjugate.

The measurement probability of the orbital angular momentum $l_z = l\hbar$ is obtained by summing the probability associated with that eigenvalue and taking the ensemble average over the turbulent aberrations [3,4]

$$P(l) = \int\iint\left\langle LG^*(r,\varphi',z)LG(r,\varphi,z)\right\rangle_{atm} rdr\frac{\exp[il(\varphi-\varphi')]}{\sqrt{2\pi}}d\varphi' d\varphi \tag{6}$$

where $\langle\bullet\rangle_{atm}$ denotes the ensemble average of turbulent atmosphere.

As in Ref. [3,4], it is assumed that the statistics of atmosphere–turbulence aberration is isotropic and since the beam profile at launch was rotationally symmetric, then the field correlation be written as

$$C_{LG}(r,\Delta\varphi,z) = \langle LG^*(r,0,z)LG(r,\Delta\varphi,z)\rangle \tag{7}$$

where $\Delta\varphi = \varphi - \varphi'$. In Eq. (6), making the substitution $\varphi = \Delta\varphi + \varphi'$, we have the orbital angular momentum measurement probability

$$P(l) = \iint\iint C_{LG}(r,\Delta\varphi,z)rdr\exp[il\Delta\varphi]d\Delta\varphi \tag{8}$$

Substituting Eq. (3) into Eq. (8) gives

$$C_{LG}(r,\Delta\varphi,z) = \left|R_{l_0,p}(r,z)\right|^2\frac{\exp(il_0\Delta\varphi)}{2\pi}\left\langle\exp\left\{-\frac{1}{2}D_s(r,\Delta\varphi)\right\}\right\rangle \tag{9}$$

where $D_S(r,\Delta\varphi) = D_S(|2r\sin(\Delta\varphi/2)|)$ is the wave structure function.

By Eq. (9), we rewrite Eq. (8) as

$$P(l) = \int_0^\infty\left|R_{l_0,p}(r,z)\right|^2\frac{1}{2\pi}\int_0^{2\pi}\exp\left[-\frac{1}{2}D_S(|2r\sin(\Delta\varphi/2)|)\right] \times\exp[-i(l-l_0)\Delta\varphi]d\Delta\varphi rdr \tag{10}$$

2.1. Turbulent model

In this paper, we consider a theoretical power spectrum model that describes non-Kolmogorov optical turbulence, which obeys a power law [14]

$$\phi_n(\kappa) = A(\alpha)\tilde{C}_n^2(z)\kappa^{\alpha} \tag{11}$$

where the term $A(\alpha)$ and $\tilde{C}_n^2(z)$ are defined by

$$A(\alpha) = \frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right),\quad 3<\alpha<4 \tag{12}$$

$$\tilde{C}_n^2(z) = \gamma C_n^2(z) \tag{13}$$

,and γ is a constant equal to unity when $\alpha = 11/3$, but otherwise has units $m^{-\alpha+11/3}$, $C_n^2(z)$ is the refractive index structure parameter of the slant channel. One of the most widely used models is the Hufnagel–Velly model described by [8,9]

$$\begin{aligned} C_n^2(z\cos\theta) = {} & 0.00594(v/27)^2\left(z\cos\theta\times10^{-5}\right)^{10}\exp(-z\cos\theta/1000) \\ & +2.7\times10^{-16}\exp(-z\cos\theta/1500) \\ & +C_n^2(0)\exp(-z\cos\theta/100) \end{aligned} \tag{14}$$

here $z\cos\theta = h$ is height of the receiver, $v = 21m/s$ is the root-mean-square wind speed, $C_n^2(0)$ is the refractive index structural characteristic of ground and θ is the zenith angle.

For non-Kolmogorov optical turbulence, the Zernike-coefficient variances are given by [15]

$$\left\langle\left|a_j\right|^2\right\rangle = \left(\frac{D}{\tilde{r}_0}\right)^{\alpha-2}\frac{(n+1)\Gamma\left(\frac{2n+2-\alpha}{2}\right)\Gamma\left(\frac{\alpha+4}{2}\right)\Gamma\left(\frac{\alpha}{2}\right)\sin\left(\pi\frac{\alpha-2}{2}\right)}{\pi\Gamma\left(\frac{2n+4+\alpha}{2}\right)}, \tag{15}$$

$n\geq1$ and $2<\alpha<4$

where $\Gamma(a)$ denotes the Gamma function, $\tilde{r}_0$ is a quantity analogous to Fried's parameter r_0 that reduces to r_0 for the case of $\alpha = 11/3$. $\tilde{r}_0 = 2.1\tilde{\rho}_0$. $\tilde{\rho}_0$ is a quantity analogous to coherence length ρ_0 that reduces to ρ_0 for the case of $\alpha = 11/3$. And $\tilde{\rho}_0$ is given by [16]

$$\tilde{\rho}_0 = \left\{\frac{2\Gamma\frac{(3-\alpha)}{2}\left(\frac{8}{\alpha-2}\Gamma\left(\frac{2}{\alpha-2}\right)\right)^{(\alpha-2)/2}}{\pi^{1/2}k^2\Gamma\left(\frac{2-\alpha}{2}\right)\int_0^z C_n'2(\xi)\left(1-\frac{\xi}{z}\right)^{\alpha-2}d\xi}\right\}^{1/\alpha-2},\quad 3<\alpha<4 \tag{16}$$