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PHOTONICS AND NANOSTRUCTURES Fundamentals and Applications

Photonics and Nanostructures – Fundamentals and Applications 12 (2014) 9–15

www.elsevier.com/locate/photonics

Invited Paper

A perfect lens for ballistic electrons: An electron-light wave analogy☆

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Received 25 July 2013; received in revised form 13 August 2013; accepted 25 August 2013
Available online 5 September 2013

Abstract

The analogy between electromagnetic waves and ballistic electrons within the Kane's model is developed and subsequently applied to a theoretical description of a quantum version of a metamaterial planar lens. Restrictions imposed on the perfect lens and the poor man's lens by available semiconductor band structures are discussed. A realistic implementation is proposed for the quantum poor man's lens, which uses specific properties of the HgTe compound. The properties of the lens are presented on the basis of a calculated transmission of oblique electrons through the lens structure.

Keywords: Metamaterial; Perfect lens; Electron lens; Kane's model; Semiconductor heterostructure

1. Introduction

Analogies between light waves and electron waves have a long history, starting in the early years of quantum mechanics [1] and culminating in extensive review papers [2,3] during the 1990s. At that time it was already well understood that there is an almost exact analogy between electromagnetic plane waves and quantum mechanical waves describing ballistic non-relativistic electrons. In fact, as present-day semiconductor techniques allow for precise monolayer growth, ballistic transport is routinely observed, and many semiconductor devices based on this analogy have been developed and experimentally tested. As representative examples we mention electrostatic lenses [4,5], prisms [6], directional couplers [7,8], filters [9] and circuit theory concepts [10]. Unfortunately, developments in this field peaked well before the emergence of metamaterials, i.e. composite materials offering electromagnetic properties not found in natural substances, such as negative permittivity and permeability [11,12].

Metamaterials have brought important new concepts into classical electromagnetism, e.g. the perfect lens [13] and transformation optics [14,15], while their quantum analogies have not received the attention that they deserve, with the exception of a few pioneering studies that will be briefly reviewed. In an early attempt [16], a particularly simple form of the analogy of an electromagnetic plane wave and an electron wave was presented. The author then transferred the idea of complementary media into the electron domain using transmission matrix formalism, and proposed the use of

☆ The article belongs to the special section Metamaterials.

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1569-4410/$ – see front matter
http://dx.doi.org/10.1016/j.photonics.2013.08.005

the complementary medium layer to improve the scanning tunneling microscopy of specific structures. Subsequently [17], this analogy was used to explore the *I–V* characteristics and the traversal times of ballistic electrons propagating normally to the boundaries of the heterostructure analogous to the metamaterial perfect lens. The electron analogy of a perfect lens was further proposed in the form of a p–n junction on a graphene sheet [18]. Next [19], it was shown that spatial transformations leading to transformation optics and to metamaterial cloaking can also be used in a very similar manner used on the Schrödinger equation. Later [20], it was shown that ballistic electrons propagating in the HgTe–CdTe heterostructure can exhibit perfect tunneling, a phenomenon largely responsible for the unique properties of the perfect lens. Important papers [21,22] then showed that envelope approximation, commonly used for describing ballistic electrons in semiconductor heterostructures, is equivalent to the effective medium theory commonly used for describing of electromagnetic metamaterials. By means of this effective medium, a perfect lens made of graphene [21] was proposed. Lastly, the analogies mentioned above were used for a study of the cloaking of matter waves [23–25].

In this paper, we will discuss the quantum analogy of the perfect lens [13]. For this purpose, the analogy between plane waves and ballistic electrons for normal incidence [20] will be extended to obliquely incident electrons. The quantum description of a semiconductor will be based on the 4-band Kane's model, the parameters of which are fixed by microscopic pseudopotential calculations. The demands on the semiconductor band structure for achieving perfect lensing are discussed, and finally we propose realistic devices using HgTe, a semiconductor with an inverted band structure.

During the final processing of the present text a paper [26] appeared, addressing the same problem. Interestingly, however, the perspective and the methodology is considerably different and the two papers complement each other rather than duplicate.

2. An analogy between electron waves and light waves

Before getting to the actual lens design, a formal analogy between ballistic electrons and electromagnetic plane waves has to be established. The topology of a perfect lens [13] is represented by a layered isotropic medium (stacking along the z-axis is assumed) in which oblique plane waves with the wavevector $\mathbf{k} = k_z\mathbf{z}_0 + k_y\mathbf{y}_0$ (k_y being the transversal wavenumber) propagate. In the quantum domain, such a heterostructure is commonly described by the 4-band Kane's model [27], where these bands correspond to conduction electrons (symmetry Γ_6), light and heavy holes (symmetry Γ_8) and split-off holes (symmetry Γ_7). Using the previously suggested [28] approximations, which involves dropping the freespace terms for the Γ_7 and Γ_8 bands, the spin states of the conduction electrons remain degenerated even at oblique incidence, and are described by a scalar equation for the wavefunction f_c

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+\frac{\hbar^2}{2m}k_y^2+E_{\Gamma_6}-E\right]f_c(z)=0, \tag{1}$$

where

$$\frac{1}{m}=\frac{2P^2}{3}\left(\frac{2}{E-E_{\Gamma_8}}+\frac{1}{E-E_{\Gamma_7}}\right) \tag{2}$$

is the inverse of the mass of the electron inside the material, k_y is the transversal wavenumber and E is the energy of the electron. E_{Γ_6}, E_{Γ_7} and E_{Γ_8} are the band edge energies of the Γ_6, Γ_7 and Γ_8 bands. Parameter P, known as the Kane's parameter, does not depend on energy, is well defined for commonly used semiconductors, and can be obtained from experiments or, as in this paper, from microscopic calculations. It is also important to note that the validity of (1), (2) was recently confirmed [22] via the application of a homogenization technique. Eq. (1) is now advantageously rewritten [16] in matrix form

$$\frac{\partial}{\partial z}\begin{bmatrix} f_c \\ \dfrac{-i\hbar}{m}\dfrac{\partial f_c}{\partial z}\end{bmatrix} = \begin{bmatrix} 0 & i\dfrac{m}{\hbar} \\ 2i\dfrac{\left(E-E_{\Gamma_6}-\dfrac{\hbar^2}{2m}k_y^2\right)}{\hbar} & 0\end{bmatrix}\begin{bmatrix} f_c \\ \dfrac{-i\hbar}{m}\dfrac{\partial f_c}{\partial z}\end{bmatrix} \tag{3}$$

and supplemented by boundary conditions, namely [27], by the continuity of f_c and $(\partial f_c/\partial z)/m$ at all heterostructure boundaries.

Due to the vector nature of electromagnetic fields, the propagation of electromagnetic plane waves in an isotropic medium, unlike ballistic electrons, depends on polarization. For the purposes of this paper, we define (see Fig. 1) the TE wave as that characterized by k_y, k_z, E_x, H_y, H_z, $\partial/\partial x \to 0$, $\partial/\partial y \to ik_y$ and the TM wave as that characterized by k_y, k_z, H_x, E_y, E_z, $\partial/\partial x \to 0$, $\partial/\partial y \to ik_y$. The propagation of TE and TM waves in an isotropic material is described by Maxwell's equations, which can be written as