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# A theoretical model for quantum nanostructures electronic wave functions, magnetic field effects

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**Abstract**

Analytical solutions of electronic wave functions in symmetric quantum ring (QR), quantum wire (QWR) and quantum dots (QD) structures are given using a parabolic coordinates system. The solutions for low-energy states are combinations of Bessel functions. The density of states of perfect 1D QR and QWR are shown to be equivalent. The continuous evolution from a 0D QD to a perfect 1D QR can be precisely described. The sharp variation of electronic properties, related to the build up of a potential energy barrier at the early stage of the QR formation, is studied analytically. Paramagnetic and diamagnetic couplings to a magnetic field are computed for QR and QD. It is shown theoretically that magnetic field induces an oscillation of the magnetization in QR.


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*Keywords:* Density of states; Quantum dot; Quantum ring; Quantum wire; Magnetic field

Progress in growth techniques has enabled experimental scientists to obtain nano-structured semiconductor materials with various shapes and organizations such as quantum wells (QW), quantum wires (QWR), quantum rings (QR) or quantum dots (QD). Confinement results in a quantization of the electronic energies and a variation of the density of states. Optoelectronic devices based on such nanostructures are expected to show improved properties as compared to conventional devices [1–4]. Theoretical models are developed in two directions. On the one hand, multi-band simulations of the electronic properties are used. On the other hand, simple one-band models try to use symmetrical nanostructure geometries to get analytical solutions [5–7]. The shape of a QR may be continuously changed experimentally from a QD for self-assembled InGaAs quantum dots grown on GaAs substrate

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[8–10]. InAs nanostructures on InP are important for applications at the telecommunication wavelength of 1.55 μm [4]. QD [11], QWR [12] or QR [13] can also be obtained on an InP substrate depending on the growth conditions. In our previous theoretical work, asymmetric lens-like QD, QR and QWR have been described using parabolic coordinates [7]. We consider in this work symmetric QWR and disk-like quantum QD and QR. It is shown that complete analytical solutions using Bessel functions can be found for electronic states. An external magnetic field can modify the excitation spectrum of QR. This effect is related to variation of the magnetization as a function of the magnetic field.

The method used in our previous work [7] for nanostructures with flat bases is extended to symmetric QWR and disk-like quantum QD and QR with the same sets of coordinates. The parabolic set of unitless coordinates ($u$, $v$, $\vartheta$) used for the QR and QD is defined by a transformation of Cartesian coordinates ($0\leqslant u\leqslant\infty$, $0\leqslant v\leqslant\infty$ and $0\leqslant\theta\leqslant 2\pi$): $x = auv\cos(\vartheta)$, $y = auv\sin(\vartheta)$ and $Z = a(u^2 - v^2)/2$ where $a$ is the parameter of the parabolic metric [6]. The volume of a symmetric QD is given by the intersection of two confocal parabolas ($u = 1$, $v = 1$) rotated around the $z$-axis, and a symmetric QR corresponds to the intersection of four parabolas ($u = u_e$, $v = u_e$, $u = 1$, $v = 1$) (Fig. 1, inset). The $a$ parameter represents either the external radius for the QD and QR, or the half-width for a QWR. A dimension less area $S$ is defined by $\mathbf{S} = \mathbf{S}_r/\mathbf{a}^2 = \frac{2}{3}(1 - \mathbf{u}_e)(1 - \mathbf{u}_e^3)$, where $S_r$ is the actual area of the cross-section (Fig. 1). The Hamiltonian is given by the expression:

$$H = -\frac{\eta^2}{2a^2(u^2+v^2)}\left[\frac{1}{u}\frac{\partial}{\partial u}\frac{u}{m_{(u,v)}}\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\frac{v}{m_{(u,v)}}\frac{\partial}{\partial v}\right] - \frac{\eta^2}{2a^2u^2v^2m_{(u,v)}}\frac{\partial^2}{\partial\theta^2} + V(u,v).$$

In the case of an infinite potential barrier, the Hamiltonian is separable in $u$, $v$ and $\vartheta$ coordinates and the wave function appears as a product of functions of independent variables $\Xi(u,v,\theta) = f(u)g(v)\mathrm{e}^{im\theta}$. The $f$ and $g$ functions are solutions of two coupled differential equations with a separation constant $C$. Solutions of these equations include confluent hypergeometric functions of the first kind $\phi$ and the second kind $\Psi$. If $C = 0$, the solutions of the problem contain simple Bessel functions

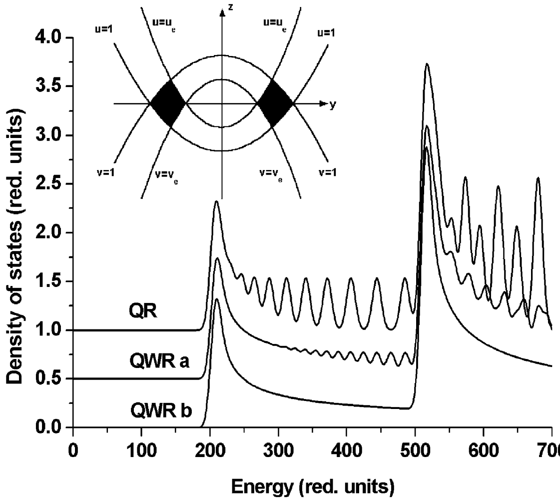


Fig. 1. Representations of the electronic densities of states for a QR (top), a QWR-a with the same volume (a) and an infinite QWR (b) ($u_e = 0.75$). In inset, projection of the parabolic coordinate surfaces in the ($y$, $z$) plane. If four orthogonal confocal parabolas are used ($u = u_e$, $v = v_e$ ,$u = 1$, $v = 1$) a symmetrical ring-shaped volume is obtained. (cross-sections of the ring are represented by dark areas).

$$f(u) = F(u,0,E,m) = \lambda_f J_{m/2}\left(\frac{\sqrt{E}u^2}{2}\right) + \mu_f K_{m/2}\left(\frac{\mathrm{i}\sqrt{E}u^2}{2}\right),$$

where $\lambda_f$ and $\mu_f$ are two constants and $E = E_r/E_\infty$ is a dimension less energy, $E_r$ is the real energy and $E_\infty = \eta^2/2ma^2$. The constants $\lambda_f$ and $\mu_f$ are defined for the QR by the boundary conditions: $F(1,C,E,m) = 0$ and $F(u_e,C,E,m) = 0$ [7]. If $u_e = 0$ (QD case), $\mu_f = 0$. The same analytical form is obtained for the $g(v)$ function.

A QWR with the same symmetric cross-section (Fig. 1) is defined from the intersection of the same parabolas in the parabolic cylindrical coordinates system: ($0\leqslant u\leqslant\infty$, $-\infty\leqslant v\leqslant\infty$): $x = auv$ and $y = a(u^2-v^2)/2$ and $Z = Z$. The wave function appears as a product of functions of independent variables $\Xi(u,v,z) = f(u)g(v)h(z)$. Solutions of the equations for the $f$ and $g$ functions contain two confluent hypergeometric functions of the first