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Time varying spatio-temporal covariance models

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ABSTRACT

In this paper, we introduce valid parametric covariance models for univariate and multivariate spatio-temporal random fields. In contrast to the traditional models, we allow the model parameters to vary over time. Since variables in applications usually exhibit seasonality or changes in dependency structures, the allowance of varying parameters would be beneficial in terms of improving model flexibility. Conditions in constructing valid covariance models and discussions on practical implementation will be provided. As an application, a set of air pollution data observed from a monitoring network will be modeled. It is found that the time varying model performs better in prediction compared with the traditional models.

1. Introduction

Spatial or spatio-temporal models have been proven to be useful in a variety of fields including environmetrics, hydrology, economics, among many others. One of the most important parts in spatial and spatio-temporal analysis is the modeling of the covariance function. A function is said to be a covariance function if the matrix defined by the covariance function is valid for all finite sets of locations and times. A covariance matrix is said to be valid if and only if it is positive semi-definite (p.s.d.). Recall that a matrix $\boldsymbol{A}$ is said to be p.s.d. if and only if $\boldsymbol{a}^\top \boldsymbol{A}\boldsymbol{a} \geq 0$ for all vectors $\boldsymbol{a}$ of comfortable dimensions. One way to guarantee the positive semi-definiteness is to define the covariance matrix based on some

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positive definite functions and make use of the celebrated Bochner's theorem (Bochner, 1955), see also Chilès and Delfiner (1999, Ch. 2). Over the past few decades, many authors introduced different kinds of valid parametric covariance models. For spatial models, the covariance is usually modeled in the form of $\mathrm{Cov}(X(\boldsymbol{s}_1), X(\boldsymbol{s}_2))$ where $\boldsymbol{s}$ is the location where X is observed. Interested readers may consult Cressie (1993) and Finkenstädt et al. (2007) for further details. For spatio-temporal models, the situation is more challenging. Traditionally, scholars built valid spatio-temporal covariance models based on the assumption that the spatial and time components are separable. Recall that a spatio-temporal covariance model is called separable if $\mathrm{Cov}(X(\boldsymbol{s}_1, t_1), X(\boldsymbol{s}_2, t_2))$ can be written as $C_S(\boldsymbol{s}_1, \boldsymbol{s}_2)C_T(t_1, t_2)$, a product of a purely spatial covariance function C_S and a purely temporal covariance function C_T. A review regarding separable models can be found in Kyriakidis and Journel (1999) and an application of separable model can be found in Rodríguez-Iturbe and Mejía (1974). The main drawback of separable models is the disallowance of the space–time interaction which leads to undesirable properties in some occasion. Hence, literature concerning non-separable covariance models appeared. Cressie and Huang (1999) introduced some classes of valid non-separable spatio-temporal covariance models. Based on the results of Cressie and Huang (1999), Gneiting (2002) introduced other classes of valid models based on completely monotonic functions. Other works include De Iaco et al. (2002), Stein (2005) and Fuentes et al. (2008), among many others. For non-separable anisotropic (depending on the directions) spatio-temporal models, see Porcu et al. (2006). We note that under our approach, the resulting spatio-temporal covariance is non-separable in general.

In the above works, in order to achieve validity, covariance parameters were assumed to be fixed both spatially and temporally. But such an assumption is clearly unnecessarily restrictive. Relaxation of the constant parameter assumption will surely be beneficial since it enhances the model flexibility. Under the purely spatial setting, Gelfand et al. (2003) attempted to include spatially varying coefficients in their models under the Bayesian framework. For the multivariate spatial settings, some details can be found in Gelfand et al. (2003, 2004) and Kleiber and Genton (2013). Our work is closely related to Kleiber and Genton (2013). In Kleiber and Genton (2013), they introduced the spatial covariance models for multivariate spatial processes which are spatial varying. Analogously, one could consider spatio-temporal processes as multivariate spatial processes, with each time point regarded as a component from the multivariate process. The temporal correlation in our work can be analogous to the cross-covariance correlation in their work. Nevertheless, we must emphasize the difference between our work and Kleiber and Genton (2013). First, forecasting in time, which cannot be done in their models, can be easily done under our proposed models. Second, in Kleiber and Genton (2013), estimation of parameters were done under non-parametric methods. In our work, full parametric methods will be employed. Under full parametric methods, predictions can be done using classical methods. In later parts, we will compare the time varying models with an ordinary separable model in terms of the predictive powers.

The rest of this paper is organized as follows. In Section 2, details for the univariate case are provided while the results for the multivariate case are given in Section 3. In Section 4, the empirical coverage rates of confidence intervals are assessed via a simulation study. In Section 5, we apply the models to a set of trivariate air pollution data recorded in California. Conclusions and discussions are provided in the last section.

2. Univariate time varying spatio-temporal covariance models

2.1. Main results

Consider the spatio-temporal random process

$$\boldsymbol{X} = \left(X\left(\boldsymbol{s}_1, t_1\right), X\left(\boldsymbol{s}_2, t_1\right), \ldots, X\left(\boldsymbol{s}_m, t_1\right), \ldots, X\left(\boldsymbol{s}_1, t_T\right), \ldots, X\left(\boldsymbol{s}_m, t_T\right)\right)^\top$$

containing time series of length T at each of the m locations. In practice the sites $\boldsymbol{s}_i \in \mathbb{R}^d$ for $d \leq 3$. We focus on the modeling of the covariance of $\boldsymbol{X}$ and therefore, throughout the whole work, it is assumed that the mean of $\boldsymbol{X}$ is $\boldsymbol{0}$. Note that the assumption is not restrictive since in practice one can always subtract the original data by the sample mean to remove the mean component. The main objective here is to introduce valid temporally varying spatio-temporal covariance models which are