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Modeling asymptotically independent spatial extremes based on Laplace random fields

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ABSTRACT

We tackle the modeling of threshold exceedances in asymptotically independent stochastic processes by constructions based on Laplace random fields. Defined as mixtures of Gaussian random fields with an exponential variable embedded for the variance, these processes possess useful asymptotic properties while remaining statistically convenient. Univariate Laplace distribution tails are part of the limiting generalized Pareto distributions for threshold exceedances. After normalizing marginal distributions in data, a standard Laplace field can be used to capture spatial dependence among extremes. Asymptotic properties of Laplace fields are explored and compared to the classical framework of asymptotic dependence. Multivariate joint tail decay rates are slower than for Gaussian fields with the same covariance structure; hence they provide more conservative estimates of very extreme joint risks while maintaining asymptotic independence. Statistical inference is illustrated on extreme wind gusts in the Netherlands where a comparison to the Gaussian dependence model shows a better goodness-of-fit. In this application we fit the well-adapted Weibull distribution, closely related to the Laplace distribution, as univariate tail model.

1. Introduction

Extreme value analysis provides a toolbox for modeling and estimating extreme events in univariate, multivariate, spatial and spatiotemporal processes (Coles, 2001; Beirlant et al., 2004;

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Davison et al., 2012). Principal objectives of spatial extreme value analysis are the spatial prediction of extremes and the extrapolation of return levels and periods beyond the historically observed range of data. A major distinction of dependence types can be made between asymptotic dependence when $\lim_{u\uparrow 1} \mathrm{pr}(F_{X_2}(X_2) \geq u \mid F_{X_1}(X_1) \geq u) > 0$ for two random variables $X_1 \sim F_{X_1}$ and $X_2 \sim F_{X_2}$ and asymptotic independence when the limit is 0, provided the limit exists. Asymptotic independence can arise in environmental and climatic data for space lags or time lags when the most extreme events become more and more isolated in time, space or space–time. For many processes like wind gust speed or heavy rainfall such behavior seems plausible owing to physical limits, and it is corroborated by empirical findings (Davison et al., 2013; Thibaud et al., 2013; Opitz et al., 2015). In this paper, our objective is to construct asymptotically independent spatial processes that are flexible and tractable models with useful properties for modeling threshold exceedances.

Models for asymptotically independent extremes must adequately capture the joint tail decay rates in multivariate distributions. A first in-depth analysis of joint tail decay was given by Ledford and Tawn (1996, 1997). Closely related bivariate models (Ramos and Ledford, 2009) provide flexibility in the joint tail, yet an explicit definition of the probability density cannot be given when only one component is extreme and the generalization to higher dimensions suffers from the curse of dimensionality. A more flexible characterization of multivariate tail behavior was developed by Wadsworth and Tawn (2013). Useful models pertaining to this framework are obtained by inverting max-stable processes (Wadsworth and Tawn, 2012), allowing for composite likelihood inference. Another approach that spans both asymptotically dependent and asymptotically independent data is presented by Wadsworth et al. (2014), who model bivariate tails by assuming independence among the radial and the angular variable in a pseudo-polar representation.

For lack of a unified theory of asymptotic independence, a variety of modeling approaches have proven useful in practice. They usually suffer from at least one of the following restrictions: joint tail decay rates are difficult to characterize; standard inference methods like classical likelihood are not available; only bivariate models are tractable and useful; the generalization to the infinite-dimensional, spatial setup is not possible; the univariate tail models prescribed by extreme value theory do not directly appear as marginal distributions in the model, necessitating marginal pretransformations that are not natural in the extreme value context.

In the following, we present the novel Laplace model for multivariate and spatial extremes. It provides a good compromise with respect to the aforementioned potential shortcomings. It is parametrized by a covariance function and closely related to standard Gaussian processes based on embedding an exponentially distributed variable for the variance. The resulting univariate distributions are of the Laplace type, and the univariate tails correspond to valid generalized Pareto limits of threshold exceedances. Classical likelihood inference using threshold exceedances is straightforward. Joint tail decay rates and conditional distributions can be characterized in various ways. We use the terminology of *spatial processes* for simplicity's sake, but an extension to the spatiotemporal context is possible through spatiotemporal covariance functions. The notion of a multivariate threshold exceedance is not uniquely defined; here we will concentrate on four sensible choices for extreme value analysis: exceedances observed at one fixed site s, exceedances of a linear combination of values at D fixed sites, or exceedances of either the maximum or minimum value over D fixed sites.

Section 2 gives a short exposition of some aspects of classical extreme value theory that are necessary to understand univariate tail models, their link to standard Laplace marginal distributions and the notion of asymptotic independence. Section 3 treats definition and inference of the asymptotically independent Laplace model for threshold exceedances, whose tail behavior is characterized and contrasted with the asymptotic dependence case. An application to modeling of spatial wind gust extremes in the Netherlands in Section 4 is followed by the concluding remarks in Section 5.

2. Extreme value theory

Presentations going beyond this short account can be found in Resnick (1987), Beirlant et al. (2004) and de Haan and Ferreira (2006).