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Bayesian spatial binary classification

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ABSTRACT

In analyses of spatially-referenced data, researchers often have one of two goals: to quantify relationships between a response variable and covariates while accounting for residual spatial dependence or to predict the value of a response variable at unobserved locations. In this second case, when the response variable is categorical, prediction can be viewed as a classification problem. Many classification methods either ignore response-variable/covariate relationships and rely only on spatially proximate observations for classification, or they ignore spatial dependence and use only the covariates for classification. The Bayesian spatial generalized linear (mixed) model offers a tool to accommodate both spatial and covariate sources of information in classification problems. In this paper, we formally define spatial classification rules based on these models. We also take a close look at two of these models that have been proposed in the literature, namely the probit versions of the spatial generalized linear model (SGLM) and the Bayesian spatial generalized linear mixed model (SGLMM). We describe the implications of the seemingly slight differences between these models for spatial classification and explore the issue of robustness to model misspecification through a simulation study. We also provide an overview of alternatives to the SGLM/SGLMM-based classifiers and illustrate the various methods using satellite-derived land cover data from Southeast Asia.

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1. Introduction

Prediction of unobserved binary or categorical variables can be cast as a classification problem, where a classification rule is used to assign an unobserved variable to a class, or category, based on a collection of observed *inputs* (e.g., predictors or covariate information). A classification rule is determined by a *decision function*, or a function of the inputs, which can be derived from either an underlying statistical model (e.g., logistic regression and discriminant analysis) or an algorithmic method such as support vector machines (SVM) and k-nearest neighbors (kNN) (see Hastie et al., 2001, for an overview). In this paper, we consider the spatial classification problem. That is, we seek to define classification rules to assign an unobserved variable associated with a spatial location (a particular point in a continuously-indexed spatial domain or an area in a discretely-indexed spatial domain) to one or more discrete classes. We refer to this spatial location as the *focal location* and the area surrounding it as the *neighborhood* of the focal location.

In classification problems involving spatially-referenced variables, often neighboring values of the unobserved/unknown variable should be used as inputs to the decision function, along with other inputs associated with the focal location and its neighbors. For example, binary or categorical images derived from satellite remote sensing often contain unobserved locations (or, "pixels" in this setting) due to errors in processing the raw data or measurement complications such as cloud cover. In these situations, formal classification methods are needed to assign values to the unobserved location so that the images can be used for various purposes in scientific investigations. While values of inputs associated with the focal location (e.g., land cover) may contain valuable information, knowledge of the class of neighboring locations may also be useful in classifying the focal location correctly. As we will illustrate, classification rules that rely on neighboring observations can be derived from the Bayesian spatial generalized linear and generalized linear mixed models (SGLMs and SGLMMs, respectively).

Some spatial classification methods have been proposed in the literature, many of which were motivated by remote sensing applications where the measured spectra serve as inputs/covariates. Unlike the classifiers derived from SGLM/SGLMMs in which spatial proximity explains the patterning of a categorical outcome after accounting for covariates (i.e., spatial dependence is in the 'residuals'), these alternative spatial classifiers take advantage of spatial dependence in the covariates associated with each location. For example, Switzer (1980) and Mardia (1984) build on the traditional linear discriminant analysis by augmenting the covariates associated with the focal location with the covariates of neighboring locations in determining classification rules. Building on this idea, Šaltytė Benth and Dučinskas (2005) and Batsidis and Zografos (2011) explicitly model the strength of spatial dependence in the covariates to guide the selection of the spatial extent and weighting of neighboring covariate values to be used in the classification rule. In cases where existing spatial classification methods do make use of the class of neighboring locations (Klein and Press, 1992; Press, 1996), the dependence is not directly modeled. Instead, the classes of neighboring locations are used to select the spatial extent of neighboring covariate values that are used as inputs to the classification rule. These final two methods, to our knowledge, are also the only existing Bayesian spatial classifiers. These spatial approaches to classification have clear utility in remote sensing applications when entire scenes (images) are completely unobserved and spectra (covariates) are often strongly informative and exhibit strong spatial dependence. However, in cases where only some pixel classes are missing, SGLM/SGLMM-based classifiers, which make direct use of both neighboring class information and covariates, is desirable.

The primary goals of this paper are to formally define spatial binary classifiers based on the probit versions of the SGLM and SGLMM and compare these two classifiers in terms of the complexity of the underlying model and the robustness to misspecification of the underlying model. As we discuss below, the probit SGLM and SGLMM have been used in the literature to model spatially-dependent binary data. However, we are not aware of existing studies exploring model misspecification in the classification/prediction setting and thus this discussion is the primary contribution of this paper. The formalism we introduce to define spatial binary classifiers based on the SGLM/SGLMM allows us to readily compare the performance of these methods to other spatial and non-spatial binary classifiers, a secondary goal of the paper. We provide a comprehensive review of these alternative methods in Appendix A.