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Classification of points in superpositions of Strauss and Poisson processes

Claudia Redenbach [a,*], Aila Särkkä [b], Martina Sormani [a,c]

[a] University of Kaiserslautern, Mathematics Department, 67653 Kaiserslautern, Germany

[b] Chalmers University of Technology and University of Gothenburg, Department of Mathematical Sciences, 412 96 Gothenburg, Sweden

[c] Fraunhofer Institut für Techno- und Wirtschaftsmathematik, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany

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ABSTRACT

Consider a realisation of a point process which is formed as a superposition of a regular point process, here a Strauss process, and some Poisson noise. The aim of the current work is to decide which of the two processes each point belongs to. We construct an MCMC algorithm which estimates the parameters of the superposition model and obtains posterior probabilities for each point of being a Strauss point. The algorithm is evaluated in a simulation study. Finally, it is applied to our motivating data set containing the locations of air bubbles, some of which are noise, in an Antarctic ice core.

1. Introduction

Superpositions of spatial point processes are not widely studied. Most of the studies in the literature consider the problem of minefield detection, where the aim is to first detect the minefield and then to classify the points in the minefield into mines and noise. The observation window is typically divided into two parts, the minefield as a region with a higher intensity containing both mines and noise and a low intensity area containing only noise. There are several approaches to this problem. Usually, it is assumed that the data are generated by two Poisson processes with different

* Corresponding author. Tel.: +49 631 205 3620; fax: +49 631 205 2748.
E-mail addresses: redenbach@mathematik.uni-kl.de (C. Redenbach), aila@chalmers.se (A. Särkkä), martina.sormani@itwm.fraunhofer.de (M. Sormani).

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intensities (see Allard and Fraley, 1997; Byers and Raftery, 1998; Stanford and Raftery, 2000; Dasgupta and Raftery, 1998). For the classification of the points on a minefield into mines or noise, Bayesian approaches have been suggested to derive the posterior probability of being a mine for each point (Cressie and Lawson, 2000; Walsh and Raftery, 2002).

The work presented in this paper is motivated by a problem from glaciology. Several deep ice cores were drilled through the Antarctic and Greenlandic ice sheets within the last decades. Ice extracted from a certain depth within the ice sheet contains a system of isolated air bubbles. The analysis of the gas decomposition in these air bubbles has become an important tool for deriving climate information from the past. A thorough interpretation of these ice core records requires an accurate dating of the ice. However, dating techniques have not developed in a satisfying way until recently and no absolute dating tools are available for polar ice. Recent dating relies on models whose key element is the simulation of the individual history of ice deformation for each specific core site. In Redenbach et al. (2009) it was shown that information on the motion history of the ice sheet can be derived from the point process of bubble centres extracted from tomographic images of ice core samples. As the bubble centres form a regular point process, motion parameters can be determined by detecting anisotropies in the neighbourhood structure of the points. Recently, it was discovered that ice core samples may contain noise bubbles which form due to relaxation of the ice after the core is taken out of the drilling hole (Weikusat et al., 2012). These bubbles do not carry any information on the motion history of the ice sheet. Hence, noise bubbles should be detected and removed prior to the motion analysis. Our assumption is that the centres of these noise bubbles are a realisation of a stationary Poisson process.

Motivated by the ice application we investigate the following problem: consider a realisation of a point process which is formed as a superposition of a regular point process, a Strauss process in our case, and some Poisson noise. The aim of the current work is to classify the points according to which of the two processes they belong to. Our work is based on the ideas presented in Walsh and Raftery (2002), where mines located in linear patterns mixed with Poisson noise are considered. To adapt the method to the model assumptions suitable for the bubble data, the linear patterns are replaced by a regular Strauss process.

Although our motivating example is a three-dimensional point pattern, we will formulate our models in arbitrary dimensions. Preliminary analyses will be performed on simulated 2D data which makes visual investigation of the results much easier. For simplicity, we will restrict attention to stationary and isotropic point processes.

The paper is organised as follows. First, we specify the superposition model in Section 2 and present the MCMC approach in Section 3. Then, in Section 4 we present the results of a simulation study, where we first assume that the superposition model parameters are known and only perform the classification, and then, estimate the model parameters simultaneously with the classification. Finally, the MCMC approach is applied to the ice data in Section 5.

2. Superposition models

2.1. Model specification

We assume that we observe a point pattern y consisting of n points contained in a study region $A \subset \mathbb{R}^d$. The point pattern y is interpreted as a superposition of two point patterns y_0 and y_1 consisting of n_0 and n_1 points, respectively. Pattern y_0 is assumed to be a realisation of a Poisson process Φ_0 while y_1 is a realisation of a Strauss process Φ_1. The two processes are assumed to be stationary, isotropic, and independent. In applications we will assume that Φ_1 models the locations of interest, e.g. the locations of bubble centres in the ice in our motivating example, while Φ_0 gives the locations of the noise points.

The distributions of both Φ_0 and Φ_1 are determined by densities with respect to the distribution of a Poisson process with unit intensity. The Poisson process Φ_0 has the density

$$g(y_0) = e^{-(\lambda_0 - 1)|A|} \lambda_0^{n_0},$$