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Fractional bivariate exponential estimator for long-range dependent random field

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ABSTRACT

Estimating long-range dependence parameter of a random field, which provides a measure for the extent of long-range dependence, is a challenging problem. Fractional Bivariate EXponential (FBEXP) estimator is proposed for long-range dependence parameter estimation in random fields observed on a regular lattice. FBEXP is a generalized version of the commonly used FEXP estimator in time series. FBEXP estimator belongs to the class of broadband semiparametric estimators, which makes it free from the optimal bandwidth selection problem, present in other semiparametric estimators. The behavior of FBEXP estimator depends on the model order of bivariate exponential model. Three data-driven model order selection criteria are introduced to serve as a guide in appropriate choice of model order for efficient estimation of long-range dependence using FBEXP. The finite sample performance of the FBEXP estimator and model order selection criteria are illustrated via simulation study. FBEXP estimator provides an efficient estimate for the long-range dependence parameter, especially when the spectral density is sufficiently smooth everywhere except at the points of singularities. Total column ozone amounts in Europe and Mediterranean region illustrate the applicability of FBEXP estimator in providing efficient estimates for direction specific long-range dependence parameters and the spectral density of the process.

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1. Introduction

Long-range dependence in random fields is emerging as an important area of statistical applications, including agricultural field trials, spatial variability in soil properties, and air ozone concentration (Dobrushin and Major, 1979; Ivanov and Leonenko, 1989; Leonenko, 1999; Doukhan et al., 2003). A random field is characterized as long-range dependent if its (auto)covariance function tends to zero slowly and is not absolutely summable over time-lags. Alternatively, a random field with unbounded spectral density at certain frequencies (including zero frequency) is considered to exhibit long-range dependence. These time- and frequency-domain characterizations of long-range dependence are closely related, but not equivalent (Lavancier, 2006). In this work, the frequency-domain characterization is employed to develop an estimator for long-range dependence parameter of a random field.

The spectral density of time series with long-range dependence is of the form,

$$f(\lambda) = |1 - \exp(-i\lambda)|^{-2\alpha} f^*(\lambda); \quad \lambda \in [-\pi, \pi], \tag{1}$$

where $\alpha \in [-0.5, 0.5]$ is the long-range dependence parameter and $f^*(\lambda)$, a smooth function bounded above and away from the zero frequency, is the short-range dependence component. A common scientific goal is to estimate the long-range dependence parameter α, which primarily governs the behavior of spectral density at some unbounded frequencies. The class of parametric estimators for α, which are typically developed in the time-domain, begin with the assumption that $f^*(\lambda)$ is a realization from an underlying stochastic model. Least squares, maximum likelihood or Whittle likelihood methods are then employed to estimate long-range dependence parameter, α and a finite set of model parameters associated with the stochastic model considered for the short-range dependence component $f^*(\lambda)$. This includes estimators developed by Granger and Joyeux (1980), Hosking (1981), Dalhaus (1989), Fox and Taqqu (1986) and Giriatis and Surgailis (1990). Empirical evidence or knowledge from an expert might be available to suggest a parametric model for $f^*(\lambda)$. However, if enough knowledge is not available to warrant such a parametric formulation then the concern of model misspecification naturally arises. A more flexible class of estimators is the semiparametric approach, which is largely based in the frequency-domain. This includes methods discussed by Geweke and Porter-Hudak, 1983; Künsch, 1987; Robinson, 1995; Hurvich et al., 1998; Hurvich and Brodsky, 2001; Moulines and Soulier, 1999. The methods under semiparametric class consider a nonparametric estimation approach for $f^*(\lambda)$, as data are not assumed to be generated from a known parametric model. Semiparametric estimators have the advantage that under some conditions (as stated below), they are consistent without the need to correctly specify a parametric model and they also avoid drawbacks resulting from model misspecification. These estimators can be classified as either narrowband or broadband, depending on whether the behavior of $f^*(\lambda)$ is modeled only near the zero frequency or over all Fourier frequencies, respectively. In narrowband estimators the consistency can be achieved as long as $(1/m + m/n)$ converges to zero, where m is the number of periodogram ordinates used by narrowband estimators for a time series of length n. The convergence rate for narrowband estimators is limited to n^{-m} with $m \leq 4/5$, irrespective of the regularity in $f^*(\lambda)$. Thus, for a sufficiently smooth $f^*(\lambda)$ a potentially misspecified parametric model can easily outperform them. The broadband estimators avoid this drawback by including all non-zero Fourier frequencies while modeling $f^*(\lambda)$ and their convergence rate depends on the regularity of $f^*(\lambda)$. A popular broadband semiparametric estimator is Fractional EXPonential (FEXP), proposed independently by Hurvich and Brodsky (2001) and Moulines and Soulier (1999). In FEXP, $f^*(\lambda)$ is modeled in terms of a finite order exponential model as $\log f^*(\lambda) = \sum_{k=0}^{h} \theta_k \cos(k\lambda)$, with real constants θ_k and model order $h \in \mathbb{Z}^+$. The consistency of FEXP estimator can be achieved when $(1/h + h/n)$ converges to zero. For a sufficiently smooth $f^*(\lambda)$, this estimator converges at a rate of $n^{-1}\log(n)$ even when the FEXP model does not hold for spectral density of the time series process. A comprehensive review for long-range dependent time series is given by Beran (2010).

Extension of long-range dependence estimators from time series to their higher-dimensional counterparts is not exactly straightforward due to the non-existence of a unique definition for long-range dependent random fields. In recent years, estimators have been proposed for long-range dependence parameter(s) in random fields. Existing estimators can be classified as either