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Isotropy test for spatial point processes using stochastic reconstruction

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ABSTRACT

We develop a model-free isotropy test for spatial point patterns. The proposed test statistic assesses the discrepancy between the uniform distribution and the empirical normalised reduced second-order moment measure of a sector of increasing central angle. The null distribution of the test statistic is approximated by the empirical distribution obtained from bootstrap-type samples, which are generated by a stochastic procedure reconstructing independent isotropic patterns that resemble the spatial structure of the given point pattern, without specifying any underlying model. Simulation studies show that, when compared with the asymptotic χ^2-test by Guan et al. (2006), the powers of the proposed test are more robust to different choices of user-chosen parameter. When applied to patterns of amacrine cells and Spanish towns, the bootstrap-type test clearly suggests rejection for the former and not rejection for the latter, while the asymptotic χ^2-test is not conclusive in either case.

1. Introduction

An often made assumption in spatial point pattern analysis is that an observed pattern is a realisation of a motion-invariant spatial point process. A spatial process is motion-invariant if it is stationary and isotropic. The distribution of a stationary process is invariant under translations while that of an isotropic process is invariant under rotations. A stationary process is not necessarily isotropic and

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a non-stationary process can be isotropic with respect to rotations about a fixed location (e.g. Byth, 1981).

Typically, stationarity and isotropy are either justified by non-statistical arguments or checked by assessing the goodness of fit of certain classes of models. However, non-statistical justification is not always unarguable and goodness-of-fit tests are often not powerful in detecting lack of fit caused by non-stationarity or anisotropy. Moreover, any lack of fit of a stationary isotropic model cannot be regarded as evidence against stationarity or isotropy. Thus, in many applications model-free tests for stationarity and isotropy would be valuable.

For stationarity, Guan (2008) and Chiu and Liu (2013) developed model-free asymptotic tests, based on the discrepancies between observed and expected numbers of points in expanding rectangular regions within a rectangular sampling window. When applying the tests to the longleaf pine data of Platt et al. (1988), which showed no strong evidence of lack of fit if the pattern is modelled by some stationary cluster processes (Stoyan and Stoyan, 1996; Ghorbani, 2013), Chiu and Liu (2013) obtained small p-values that led to a clear rejection of stationarity. Zhang and Zhou (2014) introduced another stationarity test that can be applied to non-rectangular sampling windows.

For isotropy, Guan et al. (2006) introduced a model-free statistic comparing the sample second-order product density at user-chosen lag vectors in some prescribed directions. Using the asymptotic normality of the density estimator, they showed that the limiting null distribution of their statistic is χ^2 with r degrees of freedom, where $r + 1$ is the number of *a priori* chosen directions for comparison. However, in real applications of their test, there are several practical issues. First, their simulation results reveal that the power of the test is quite sensitive to the bandwidth for the kernel estimator of the second-order product density and to the magnitudes of the lag vectors, even if these values are chosen within their recommended ranges. Second, if one wants to compare more directions, the price to pay is a larger value for the degrees of freedom, which may lead to lower powers. Third, their statistic involves the inverse of an empirical covariance matrix, which may be ill-conditioned. Nevertheless, they offered some helpful suggestions from their experience on these issues.

Fig. 1(a) shows the point pattern formed by the locations of 69 Spanish towns in a square sampling window; see Comas et al. (2011), Delicado et al. (2010), Illian et al. (2008), Ripley (1977, 1988), and Stoyan et al. (1995). The pattern in Fig. 1(b) is a bivariate pattern of 152 and 142 points, respectively, of two different types of displaced amacrine cells in the retina of a rabbit; the "on" and "off" cells represent the types that transmit information when a light is "on" and "off", respectively; see Diggle (2013) and Illian et al. (2008). In the literature these two patterns are modelled and analysed as realisations of motion-invariant processes. For the stationarity hypothesis, applying the tests in Guan (2008) and Chiu and Liu (2013) to each pattern does not suggest rejection. For isotropy, however, visual inspection of the empirical densities $\hat{\vartheta}(t)$ of the nearest-neighbour orientation distribution (Illian et al., 2008, Section 4.5.2) in Fig. 2 clearly suggests a bimodal distribution for the patterns of the "on" and "off" amacrine cells. When we ignore the "on" or "off" marks and consider the pattern of all 294 unmarked cells, the empirical density, which is flattened a bit but whose standard error should also be reduced because of a larger sample size, still reveals a bimodal shape. For the pattern of Spanish towns, because there are only 69 points, it seems reasonable to say that the empirical density deviates not much from the uniform density.

However, when we applied the isotropy test by Guan et al. (2006) to these patterns with parameter values chosen within the recommended ranges, the results are inconclusive because the p-values, depending on the magnitude of the lag vectors and the bandwidth, range from 0.0000 to 1.0000 for the Spanish towns, from 0.0322 to 0.7216 for the unmarked amacrine cells, from 0.0029 to 0.9544 for the "on" amacrine cells, and from 0.0054 to 0.9197 for the "off" amacrine cells. Such a wide range of p-values of a given pattern shows that in practice, we need an alternative test that requires fewer user-chosen parameters and is less sensitive to the choice of the values for these parameters. As we can see in Section 5, our approach described below (which is not based on $\hat{\vartheta}(t)$), when applied to these two data sets, is able to offer robust and strong evidence to reject the isotropy hypothesis for the amacrine cells and to give consistently large p-values for the Spanish towns.

The idea of the test statistic proposed in this paper comes from the orientation analysis introduced in Ohser and Stoyan (1981), who considered the normalised reduced second-order moment measure,