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Variational Bayes approach for classification of points in superpositions of point processes

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We investigate the problem of classifying superpositions of spatial point processes. In particular, we are interested in realizations formed as a superposition of a regular point process and a Poisson point process. The aim is to decide which of the two processes each point belongs to. Recently, a Markov chain Monte Carlo (MCMC) approach was suggested by Redenbach et al. (2015), which however, is computationally heavy. In this paper, we will introduce a method based on variational Bayes approximation and compare its performance to the performance of a slightly refined version of the MCMC approach.

1. Introduction

Classification of points in a superposition of spatial point processes is very difficult but of great practical interest. Most of the studies in the literature consider the situation where the data are generated by two Poisson processes with different intensities and where minefield detection is in focus. The area of interest is typically divided into two parts, one with low intensity in which only noise is present and the other with a higher intensity containing both mines and noise

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(see Allard and Fraley, 1997; Byers and Raftery, 1998; Dasgupta and Raftery, 1998; Cressie and Lawson, 2000; Stanford and Raftery, 2000). After detecting the boundaries of the minefield, the points in this area can be classified to be either mines or noise. Typically, this classification is based on Bayesian approaches which allow to estimate the posterior probability for each point to be a mine.

There are also some studies where a regular process is superimposed with Poisson noise. Walsh and Raftery (2002) consider mines located in parallel lines mixed with Poisson noise. A test based on partial Bayes factors to test the Poisson hypothesis against Strauss superimposed with Poisson is presented by the same authors, see Walsh and Raftery (2005). Furthermore, having a glaciological problem in mind Redenbach et al. (2015) introduce an MCMC approach to classify the points in a superposition of a Strauss and a Poisson process using some of the ideas by Walsh and Raftery (2002). This method seems to classify the points quite well in the sense that Ripley's K function for the classified Strauss process does not show any significant deviation from that of the original process. In practice, the parameters of the superposition model need to be estimated simultaneously with the classification of the points. Even though the quality of the parameter estimates is not that good, the classification still seems to work.

In this paper, we will introduce an alternative approach based on variational Bayes approximation. We will restrict ourselves to superpositions of two stationary and isotropic point processes, where one of the processes is a pairwise interaction process and the other one is a Poisson process. We will compare the performance of the new variational Bayes classification approach to the performance of the MCMC approach. We will also recall a classification method based on distances to the nearest neighbours, see Byers and Raftery (1998). This method was introduced for a superposition of two Poisson processes but according to the authors, it works even when one of the processes is regular.

The paper is organized as follows. First, we recall the two methods that can be found in the literature, namely the method based on the nearest neighbour distances and the MCMC approach. Then, we will introduce the new approach based on variational Bayes approximation. Furthermore, we perform a simulation study in 2D to compare the new method to the existing ones in the case where the regular process is a Strauss process.

2. Model specification

We assume that we observe a point pattern $\mathbf{x}$ consisting of n points contained in a study region $A \subset \mathbb{R}^d$. The point pattern $\mathbf{x}$ is interpreted as a superposition of two point patterns $\mathbf{x}_0$ and $\mathbf{x}_1$ consisting of n_0 and n_1 points, respectively, and $n = n_0 + n_1$. We restrict ourselves to the case where the pattern $\mathbf{x}_0$ is a realization of a Poisson process Ξ_0 and $\mathbf{x}_1$ a realization of a regular process, here a Strauss process Ξ_1. Note, however, that the Strauss process could be replaced by any pairwise interaction Gibbs point process. Furthermore, if other than regular processes were of interest, a more general class of models could be considered. The two processes are assumed to be stationary, isotropic, and independent of each other. In applications we often assume that Ξ_1 models the locations of interest while Ξ_0 gives locations of some noise points.

The distributions of both Ξ_0 and Ξ_1 are determined by densities with respect to the distribution of a Poisson process with unit intensity. The Poisson process Ξ_0 has the density

$$f_0(\mathbf{x}_0) = e^{-(\lambda_0 - 1)|A|}\lambda_0^{n_0},$$

where $\lambda_0 > 0$ is the intensity and $|A|$ is the volume of the study region A. The density for the Strauss process Ξ_1 (conditionally on A) is

$$f_1(\mathbf{x}_1) = \alpha\beta^{n_1}\gamma^{s_r(\mathbf{x}_1)},$$

where $s_r(\mathbf{x}_1)$ is the number of pairs of points in $\mathbf{x}_1$ with distance less than or equal to r, and α is the normalizing constant (see Møller and Waagepetersen, 2004). This density depends on three parameters: $\beta > 0$ is a parameter governing the intensity, $0 \le \gamma \le 1$ is the strength of interaction, and $r > 0$ is the range of interaction. It is well-known that it is difficult to estimate γ and r simultaneously. Hence, we assume r to be known, such that the parameters of the full model $\mathbf{x} = \mathbf{x}_0 \cup \mathbf{x}_1$ are $\theta = (\lambda_0, \beta, \gamma)$. The intensity λ_1 of the Strauss process depends on the parameters β, r and γ