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Estimation of space deformation model for non-stationary random functions

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ABSTRACT

Stationary Random Functions have been successfully applied in geostatistical applications for decades. In some instances, the assumption of a homogeneous spatial dependence structure across the entire domain of interest is unrealistic. A practical approach for modeling and estimating non-stationary spatial dependence structure is considered. This consists in transforming a non-stationary Random Function into a stationary and isotropic one via a bijective continuous deformation of the index space. So far, this approach has been successfully applied in the context of data from several independent realizations of a Random Function. In this work, we propose an approach for non-stationary geostatistical modeling using space deformation in the context of a single realization with possibly irregularly spaced data. The estimation method is based on a non-stationary variogram kernel estimator which serves as a dissimilarity measure between two locations in the geographical space. The proposed procedure combines aspects of kernel smoothing, weighted non-metric multi-dimensional scaling and thin-plate spline radial basis functions. On a simulated data, the method is able to retrieve the true deformation. Performances are assessed on both synthetic and real datasets. It is shown in particular that our approach outperforms the stationary approach. Beyond the prediction, the proposed method can also serve as a tool for exploratory analysis of the non-stationarity.

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1. Introduction

In the statistical analysis of spatial processes, modeling and estimating the spatial dependence structure is fundamental. It is used by prediction techniques like kriging or conditional simulations. Its description is commonly carried out using statistical tools such as the variogram or covariogram calculated on the entire domain of interest and considered under the stationarity assumption, for reasons of parsimony or mathematical convenience. The complexity of the spatial component of the analyzed process is therefore limited.

The assumption that the spatial dependence structure is translation invariant over the whole domain of interest may be appropriate, when the latter is small in size, when there is not enough data to justify the use of a complex model, or simply because there is no other reasonable alternative. Although justified and leading to a reasonable analysis, this assumption is often inappropriate and unrealistic given certain spatial data collected in practice. Non-stationarity can occur due to many factors, including specific landscape and topographic features of the region of interest or other localized effects. These local influences can be observed computing local variograms, whose characteristics may vary across the domain of observations. For this type of non-stationary structures, making spatial predictions using conventional stationary methods is not appropriate. Indeed, applying stationary approaches in such cases would be liable to produce less accurate predictions, including an incorrect assessment of the estimation error (Stein, 1999).

Several approaches have been proposed to deal with non-stationarity through second order moments (see Guttorp and Schmidt, 2013; Sampsonet al., 2001; Guttorp and Sampson, 1994, for a review). One of the most popular methods of introducing non-stationarity is the space deformation of Sampson and Guttorp (1992) and other (Meiring et al., 1997; Perrin and Monestiez, 1998; Iovleff and Perrin, 2004). It consists in starting with a stationary Random Function, and then transforming the distance in some smooth way to construct a non-stationary Random Function. Maximum likelihood and Bayesian variants of this approach have been developed by Mardia and Goodall (1993), Smith (1996), Damian et al. (2001), and Schmidt and O'Hagan (2003). Perrin and Meiring (1999), Perrin and Senoussi (2000), Perrin and Meiring (2003), Genton and Perrin (2004) and Porcu et al. (2010) established some theoretical properties about uniqueness and richness of this class of non-stationary models. Some adaptations have been proposed recently by Castro Morales et al. (2013), Bornn et al. (2012), Schmidt et al. (2011) and Vera et al. (2008, 2009). A fundamental limitation of all estimation methodologies presented so far is the fact that implementation requires multiple independent realizations of the Random Function in order to obtain an estimated sample covariance or variogram matrix. The idea of having several independent realizations of the natural field is unrealistic because there are not multiple parallel physical worlds. In practice, the approach is feasible when a time series is collected at each location as this gives the necessary, albeit dependent, replications. In general, we would prefer to incorporate a temporal aspect in the modeling rather than attempting repairs (e.g., differencing and detrending) to achieve approximatively independent realizations. However, many geostatistical applications involved only one measurement at each site or equivalently, only one realization of a Random Function. Anderes and Stein (2008) and Anderes and Chatterjee (2009) are the first authors to address the estimation of space deformation model in the case of a single realization of a Random Function, obtained as the transformation of a Gaussian and isotropic stationary Random Function. They exhibit a methodology based on quasi-conformal mappings and approximate likelihood estimation of the local parameters that characterize the deformation derived from partitioning densely observed data into subregions and assuming independence of the Random Function across partitions. However, this approach has not been applied to real datasets and requires very dense data.

In this work, we follow the pioneering work of Sampson and Guttorp (1992), while freeing the strong assumption of replication and do not make any distributional assumptions. In addition, we take into account other shortcomings associated with this approach that are: the required property of the deformation to be bijective and the computational challenge to fit the model for moderate and large datasets. To do so, we propose an estimation procedure based on the inclusion of spatial constraints and the use of a set of representative points referred to as anchor points rather than all data points to find the deformation. The proposed method provides a non-parametric estimation of the deformation