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Measuring aggregation of events about a mass using spatial point pattern methods

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ABSTRACT

We present a methodology that detects event aggregation about a mass surface using 3-dimensional study regions with a point pattern and a mass present. The Aggregation about a Mass function determines aggregation, randomness, or repulsion of events with respect to the mass surface. Our method closely resembles Ripley's K function but is modified to discern the pattern about the mass surface. We briefly state the definition and derivation of Ripley's K function and explain how the Aggregation about a Mass function is different. We develop the novel function according to the definition: the Aggregation about a Mass function times the intensity is the expected number of events within a distance h of a mass. Special consideration of edge effects is taken in order to make the function invariant to the location of the mass within the study region. Significance of aggregation or repulsion is determined using simulation envelopes. A simulation study is performed to inform

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researchers how the Aggregation about a Mass function performs under different types of aggregation. Finally, we apply the Aggregation about a Mass function to neuroscience as a novel analysis tool by examining the spatial pattern of neurotransmitter release sites as events about a neuron.

1. Introduction

Ripley's K function is a tool used to characterize the strength of spatial dependence at multiple scales in a spatial point pattern. The K function has been widely used to identify clustering, randomness, or regularity among events in a spatial point pattern Ripley (1977). Recent applications of Ripley's K function have been to the field of biology, where the spatial organization of molecules can be described with a spatial point pattern Lagache et al. (2013); Kellner et al. (2007); Jenei et al. (2009). Ripley's K function has been used in the field of neuroscience primarily to describe the spatial pattern of neurons in 3-dimensions Jafari-Mamaghani et al. (2010); Hansson et al. (2013); Eglen and Wong (2008); Millet et al. (2011). Recent research explored distance functions to a single point reference, Joyner et al. (2014) which is similar but different than our method. This paper explores a function based on Ripley's K function to characterize the level of aggregation of a point pattern relative to a *mass*. Analysis of spatial aggregation relative to a mass is a novel application.

This work was motivated by the increasing recognition that communication between neurons depends on the fine architecture of presynaptic (neurotransmitter-releasing) and postsynaptic (neurotransmitter-sensing) elements in the brain. The spatial pattern of release sites about a neuron is of physiological significance to synaptic transmission but has not yet been described mathematically. Toward this end, image stacks of neurotransmitter-releasing objects relative to a neuron were rendered in 3-dimensions and analyzed using a function similar to Ripley's K function. If we consider the objects as events, these images can be reduced to a 3-dimensional spatial point pattern distributed around a neuron. The objective in this application is to ascertain whether the objects are aggregated about the surface of the neuron, and if so to what degree. Applying the usual Ripley's K function to this point pattern, ignoring the neuron body, would measure the spatial aggregation among the objects, but would not tell us whether these objects tend to be aggregated about neuronal surfaces. A different function other than the K function is needed to assess whether the objects are aggregated *about* a neuron.

This paper consists of five additional sections. Since the motivation for this new method is derived from Ripley's K function, Section 2 briefly reviews the construction of Ripley's K. Section 3 introduces the Aggregation about a Mass function using study regions with a point pattern and a mass. Construction of this method mimics the construction of Ripley's K, but differences are noted and discussed. Section 4 shows results of a simulation study assessing how well this new method characterizes the spatial aggregation under various known conditions. Then, by applying the Aggregation about a Mass function to neuroscience, analysis of events about a neuron is conducted in Section 5. Finally, a discussion of important topics and future studies are discussed.

2. Ripley's K function

2.1. Definition and estimation

For a homogeneous point pattern, Ripley's K function Ripley (1976, 1977) is defined by:

$$\lambda K(h) = E(\text{number of additional events within distance } h \text{ of a single arbitrary event}) \quad (1)$$

where λ = the intensity or the mean number of events per unit area, assumed constant throughout the study region $\mathcal{R}$, and E is the expectation function. To estimate $K(h)$, let d_{ij} be the distance between