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Disaggregation of spatial autoregressive processes

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ABSTRACT

An aggregated Gaussian random field, possibly strong-dependent, is obtained from accumulation of i.i.d. short memory fields via an unknown mixing density φ which is to be estimated. The so-called disaggregation problem is considered, i.e. φ is estimated from a sample of the limiting aggregated field while samples of the elementary processes remain unobserved. Estimation of the density is via its expansion in terms of orthogonal Gegenbauer polynomials. After defining the estimators, their consistency and convergence rates are discussed. An example of application to β-convergence in EU GDP per capita is discussed.

1. Introduction

Consider a random field defined on a regular rectangular lattice, the so-called doubly-geometric process, defined by the equation

$$Y_{s,t} = \theta_1 Y_{s-1,t} + \theta_2 Y_{s,t-1} - \theta_1\theta_2 Y_{s-1,t-1} + \varepsilon_{s,t}, \quad (s,t) \in \mathbb{Z}^2 \tag{1}$$

where $-1 < \theta_1, \theta_2 < 1$ and $\varepsilon_{s,t}$, $(s,t) \in \mathbb{Z}^2$ is a white noise with null mean and variance σ^2. This process was first studied by Martin (1979) with the aim of providing an easy to use spatial process, which may serve as a good approximation in many applications.

In this paper we consider the so-called disaggregation problem: starting from independent individual fields of the form (1) with random coefficients θ_1 and θ_2, a new random field is constructed by aggregating the individual fields. The aggregating mechanism, due to randomness of the θ's coefficients, originates from an unknown mixing density which is to be estimated. The task here is

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to accomplish the estimation procedure having availability of data from an aggregated field. This is a typical situation in practical problems where an observed phenomenon may arise as a contribution of a myriad of micro-phenomena.

To be more precise, consider a sequence of independent random fields $Y^{(j)}_{s,t}$, $(s,t) \in \mathbb{Z}^2$, $j = 1, 2, \ldots$, defined as

$$Y^{(j)}_{s,t} = \theta^{(j)}_1 Y^{(j)}_{s-1,t} + \theta^{(j)}_2 Y^{(j)}_{s,t-1} - \theta^{(j)}_1 \theta^{(j)}_2 Y^{(j)}_{s-1,t-1} + \varepsilon^{(j)}_{s,t}, \quad (s,t) \in \mathbb{Z}^2 \tag{2}$$

where

I. $\varepsilon^{(j)}_{s,t}$, $j = 1, 2, \ldots$ is a sequence of independent copies of zero mean and unit variance strong white noise, i.e. $\varepsilon^{(j)}_{s,t}$, $(s,t) \in \mathbb{Z}^2$ are independent in (s,t).

II. $(\theta^{(j)}_1, \theta^{(j)}_2)$, $j = 1, 2, \ldots$ are independent copies (in j) of a random vector (θ_1, θ_2) supported in $[-1, 1] \times [-1, 1]$ and satisfying

$$\boldsymbol{E} \frac{1}{1-\theta_1^2} < \infty, \qquad \boldsymbol{E} \frac{1}{1-\theta_2^2} < \infty.$$

III. The sequences $\{\theta^{(j)}_1, \theta^{(j)}_2\}_{j \geq 1}$ and $\{\varepsilon^{(j)}\}_{j \geq 1}$ are independent.

IV. It is assumed that the distribution of $(\theta^{(j)}_1, \theta^{(j)}_2)$ admits a mixture density $\varphi(\theta_1, \theta_2) = \varphi_1(\theta_1)\varphi_2(\theta_2)$ (independent case) such that

$$\int_{-1}^{1} \int_{-1}^{1} \frac{\varphi_1(\theta_1)\varphi_2(\theta_2)}{(1-\theta_1^2)(1-\theta_2^2)} \, d\theta_1 \, d\theta_2 < \infty. \tag{3}$$

An aggregated field $X_{s,t} = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} Y^{(j)}_{s,t}$, whose convergence properties will be characterized more precisely below, is constructed. In the next section, we will define an estimator of the mixing density $\varphi(\theta_1, \theta_2)$ based on data obtained from the aggregated field $X_{s,t}$. The technique of estimation is based on an expansion in terms of orthogonal Gegenbauer polynomials. This approach has been used by Leipus et al. (2006) and Celov et al. (2010) in the context of random processes. In this paper we will use a bivariate setting; however the set-up provided lends itself to a straightforward extension to a multi-dimensional setting. It will turn out that the proposed estimator of the mixing density is consistent while asymptotic normality will not follow given the presence of *edge*-effects. To avoid *edge*-effects one could use unbiased covariance estimators instead, which however have the drawback to be not always positive-definite. Alternatively, tapered covariance estimators could be used; we do not pursue this approach here as, we will see, our estimation procedure requires the choice of weighting functions; the necessity of selecting a data taper as well would unduly complicate the estimation procedure. For further discussion on these issues one can consult Dahlhaus and Künsch (1987) or Guyon (1982).

A seminal paper on aggregation of processes is that of Granger (1980) where it is also shown that accumulation of short-memory random processes can lead to long-memory macro phenomena; we will allow for the possible presence of long-memory in our development. Several papers have been successively devoted to the subject, among them we mention Davidson (1991, 2002), Zaffaroni (2004), Leonenko and Taufer (2005), Beran et al. (2010), Davidson and Monticina (2010) in the context of random processes. Extensions to random fields on a lattice have been considered by various authors, see Lavancier (2005) for a review and Lavancier (2011) for more recent developments. In our set-up note that model (1) can be seen as a special case of spatial unilateral AR models as defined by Whittle (1954), i.e.

$$Y_{s,t} = \sum_{k=0}^{p_1} \sum_{l=0}^{p_2} \theta_{kl} Y_{s-k,t-l} + \varepsilon_{s,t}, \quad \theta_{00} = 0,$$

with $p_1 = p_2 = 1$, $\theta_{11} = \theta_{01}\theta_{10}$; it has also been discussed in connection with the spatial unilateral first order ARMA model as defined by Basu and Reinsel (1993). More recently Baran and Pap (2009) consider the simpler case $Y_{s,t} = \theta_1 Y_{s-1,t} + \theta_2 Y_{s,t-1} + \varepsilon_{s,t}$, $(s,t) \in \mathbb{Z}^2$ which has a stationary solution