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Transmission and scattering properties of acoustic waves in phononic band gap materials

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Abstract

We have developed a formula for studying the transmission and scattering properties of finite-sized phononic band gap (PBG) material. We will show that based on the far field approach the transmission coefficients can be obtained by treating PBG samples as scattering objects. We find that the results agree well with the band structure.

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1. Introduction

The acoustic properties of a locally homogeneous and isotropic composite material is characterized by a set of parameters varying in space: mass density ρ, Lamé coefficients λ, and μ. In this paper we focus on the composite materials, which consist of homogeneous particles distributed periodically in a host medium and are characterized by different mass densities and Lamé coefficients. When identical particles are distributed periodically in a host medium, the composite material may be referred to as a phononic crystal. Recently, the propagation of elastic or acoustic waves in a phononic crystal has received much renewed attention [1–7]. It makes the possibility of the achievement of complete frequency band gaps that are useful to prohibit specific vibrations in accurate technologies such as transducers and sonar.

The plane-wave, the finite-difference, and the multiple-scattering methods are commonly used. To study the propagation of acoustic waves in phononic band gap materials (phononic crystal), we consider a two-dimensional periodic system consisting of finite cylinders of circular cross-section. The system is periodic in the x–y plane and within it there is a translational invariance in the direction (z) parallel to the cylinders. The intersection of the cylinders with a transverse plane makes a square lattice. We treated finite PBG samples as scattering objects in open geometry. The radiation boundary condition was naturally imposed. We have independently adopted this method to study the transmission scattering and radiation properties of finite PBG samples. In the case of transmission, considering far field approach, a generalized transmission coefficient can be defined. In terms of the far-field total scattering amplitude, we can retrieve the dispersion relations and the decay length inside a gap. We have calculated the transmission coefficient and interpreted why the results agree well with the band structure.

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2. Calculation of the transmission coefficient

The displacement vector $\vec{U}(\vec{r},t)$ in a homogeneous elastic medium of mass density ρ and Lamé coefficients λ, μ satisfies the following equation:

$$(\lambda+2\mu)\nabla(\nabla\cdot\vec{U})-\mu\nabla\times(\nabla\times\vec{U})-\rho\partial_t^2\vec{U}=0 \quad (1)$$

In the case of a harmonic elastic wave with angular frequency ω, we have

$$\vec{U}(\vec{r},t)=\mathrm{Re}[u(\vec{r})\exp(-\mathrm{i}\omega t)], \quad (2)$$

and Eq. (1) was reduced to the following time-independent form

$$(\lambda+2\mu)\nabla(\nabla\cdot\vec{u})-\mu\nabla\times(\nabla\times\vec{u})+\rho\omega^2\vec{u}=0. \quad (3)$$

Defining

$$\vec{u}=\vec{l}+\vec{m}+\vec{n}, \quad (4)$$

where

$$\vec{l}=\nabla\varphi, \quad (5)$$

$$\vec{m}=\nabla\times(\chi\vec{Z}), \quad (6)$$

$$\vec{n}=\nabla\times\nabla\times(\psi\vec{Z}), \quad (7)$$

where $\vec{Z}$ is the unit vector along the z-axis. ϕ, χ and ψ are the displacement potential functions of longitudinal and two transverse waves, respectively. The displacement potential function of the incident longitudinal waves can be expanded in terms of the cylindrical Bessel function [8]

$$\varphi_{\mathrm{inc}}=\exp(jk_z z)\sum_{-\infty}^{\infty}(j)^n J_n(k_{\mathrm{lr}}r)\exp(jn\theta), \quad (8)$$

where $k_{\mathrm{lr}}=(k_l^2-k_z^2)^{1/2}$ is the radial component of the incident wave vector; J_n is the Bessel function of the first kind of order n; k_z is the z-axis component of the incident wave vector; k_l is the longitudinal wave numbers; r is the normal distance of the field spot away from z-axis; and θ being the angle of direction.

The displacement potential functions of the longitudinal and transverse scattered waves can also be expanded:

$$\varphi_{\mathrm{sc}}=\exp(jk_z z)\sum_{n=-\infty}^{\infty}A_n H_n(k_{\mathrm{lr}}r)\exp(jn\theta), \quad (9)$$

$$\psi_{\mathrm{sc}}=\exp(jk_z z)\sum_{n=-\infty}^{\infty}B_n H_n(k_{\mathrm{tr}}r)\exp(jn\theta), \quad (10)$$

$$\chi_{\mathrm{sc}}=\exp(jk_z z)\sum_{n=-\infty}^{\infty}C_n H_n(k_{\mathrm{tr}}r)\exp(jn\theta), \quad (11)$$

where H_n is Hankel function, using the same method, we can expand the displacement potential functions of the incident transverse waves in terms of the cylindrical Bessel functions. Hence, the displacement potential functions of the incident transverse waves inside the cylinders are expanded as:

$$\varphi_{\mathrm{in}}=\exp(jk_z z)\sum_{n=-\infty}^{\infty}\{[A_n H_n(k_{\mathrm{lr}}r)+D_n\delta_{n0}] + [B_n J_n(k_{\mathrm{tr}}\cdot r)+E_n\delta_{n0}]\}\exp(jn\theta), \quad (12)$$

$$\psi_{\mathrm{in}}=\exp(jk_z z)\sum_{n=-\infty}^{\infty}\{[A_n J_n(k_{\mathrm{lr}}\cdot r)+D_n\delta_{n0}] + [B_n J_n(k_{\mathrm{tr}}\cdot r)+E_n\delta_{n0}]\}\exp(jn\theta), \quad (13)$$

$$\chi_{\mathrm{in}}=\exp(jk_z z)\sum_{n=-\infty}^{\infty}[C_n J_n(k_{\mathrm{tr}}\cdot r)+F_n\delta_{n0}]\exp(jn\theta), \quad (14)$$

where A_n–F_n are coefficients and $k_{\mathrm{tr}}=(k_t^2-k_z^2)^{1/2}$.

Following, we consider a sample of the two-dimensional periodic arrays system. The sample was made of d-radius rods with lattice constant a. The position of the rod with index j corresponds to $\vec{r}_j=(r_j,\theta_j)$. What are around this rod are incident waves involving external sources and scattered waves from other rods. The total field around this rod is $u=u_{\mathrm{inc}}+u_{\mathrm{scatt}}$. The coefficients A_n–F_n are defined depending on the boundary conditions.

In the light of the continuity of the displacements, there are

$$u_i^{\mathrm{inc}}|_{r=d}+u_i^{\mathrm{sc}}|_{r=d}=u_i^{\mathrm{in}}|_{r=d} \quad i:(r,\theta,z), \quad (15)$$

Due to the continuity of the stresses, there exists

$$P_i^{\mathrm{inc}}|_{r=d}+P_i^{\mathrm{sc}}|_{r=d}=P_i^{\mathrm{in}}|_{r=d} \quad i:(r,\theta,z), \quad (16)$$

where

$$P_i=\sum_j\sigma_{ij}n_j \quad i,j:(r,\theta,z),$$

and

$$\sigma_{ij}=2\rho c_t^2u_{ij}+\rho(c_l^2-2c_l^2)\delta_{ij}\sum_l u_{ll} \quad i,j,l:(r,\theta,z).$$

where σ_{ij} are the stress tensor elements and u_{ij} are the strain tensor elements which result from the components of the displacement vector. The superscripts inc, sc, in denote the incident, the scattered and the inner field, respectively.

In the far field, when $k_lr(k_tr)\gg1, u_{\mathrm{scatt}}(\vec{r})\to f_{\mathrm{s}}(\theta)\exp(\mathrm{i}kr)/\sqrt{r}$.

The total scattering amplitude of the longitudinal waves from Eqs. (9) to (11) is

$$f_{\mathrm{s}}(\theta)=\frac{2}{\sqrt{\pi k_l}}\left|\sum_{n=-\infty}^{n}j^{-n}A_n^N\exp(jn\theta)\right|. \quad (17)$$

For acoustic wave transmission, a slit with width w along the y direction is put between a source and the sample. Acoustic waves propagate along x direction. In this case, the incident field can be obtained from the Kirchoff integral