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Variational-based locking-free energy–momentum schemes of higher-order for thermo-viscoelastic fiber-reinforced continua

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Highlights

- New locking-free energy–momentum schemes.
- Higher-order accurate energy–momentum schemes.
- Energy–momentum schemes for anisotropy.
- Energy–momentum schemes derived from a mixed principle.

Abstract

Locking-free finite elements and energy–momentum time integration schemes are two of the best-known algorithmic improvements of finite element methods. Both are developed since the middle of the eighties of the last century, but usually independently from each other. Therefore, a smart interface between both algorithms is rarely a development goal of the researcher. In this paper, we present such a smart interface, namely the Hu–Washizu procedure applied to the principle of *virtual power*. By using of the resulting mixed variational principle, we avoid locking in a spatial finite element discretization of non-isothermal inelastic fiber-reinforced materials, and obtain a family of corresponding higher-order accurate energy–momentum schemes. Thereby, we consider volumetric locking in the matrix material and line locking in the fibers. We show that this reduction of locking in the energy–momentum schemes leads to an increase of the maximum time step size, such that the efficiency of the time integration is improved in the sense that less CPU time is required. This could be shown by using an automatic time step size control with the iteration number of the applied Newton–Raphson scheme as target function. As numerical examples, we consider slender fiber-reinforced structures as a turbine rotor and a lightweight beam consisting of fiber-reinforced trusses. Here, we simulate different combinations of mechanical and thermal Dirichlet and Neumann boundary conditions.

Keywords: Fiber-reinforced continua; Anisotropy; Energy–momentum schemes; Higher-order accuracy; Variational principle; Locking-free finite elements

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1. Introduction

In this paper, we present a locking-free *strain–stress space approximation* for the energy–momentum schemes in Ref. [1], which can be seen as on-going development of the Hu–Washizu procedure applied to the principle of virtual work (see Refs. [2–4]). For dynamic problems, we show a new alternative to the Simo–Taylor–Pister functional

$$\Pi_{\mathrm{STP}}(\boldsymbol{\varphi},\tilde{J},p) := \int_{\mathscr{B}_0} \Psi^{\mathrm{iso}}\left(\bar{\boldsymbol{C}}(\boldsymbol{\varphi})\right) \mathrm{d}V + \int_{\mathscr{B}_0} \Psi^{\mathrm{vol}}(\tilde{J})\,\mathrm{d}V - \int_{\mathscr{B}_0} p\left[\tilde{J} - \det\boldsymbol{F}(\boldsymbol{\varphi})\right]\mathrm{d}V + \Pi^{\mathrm{ext}}(\boldsymbol{\varphi}) \tag{1}$$

based on an isochoric free energy function Ψ^{iso} depending directly on the unimodular part $\bar{\boldsymbol{C}} := (\det\boldsymbol{C})^{-\frac{1}{n_{\mathrm{dim}}}}\boldsymbol{C}$ of the right Cauchy–Green tensor $\boldsymbol{C} := \boldsymbol{F}^T\boldsymbol{F}$. Thereby, $\boldsymbol{F} := \partial\boldsymbol{\varphi}/\partial\boldsymbol{X}$, with $\boldsymbol{X} \in \mathscr{B}_0$, designates the deformation gradient, and $\boldsymbol{\varphi} : \mathscr{B}_0 \to \mathscr{B}_t$ denotes the deformation mapping between the initial configuration $\mathscr{B}_0$ and the current configuration $\mathscr{B}_t$. We recall that, according to the Hu–Washizu procedure, Ψ^{vol} depends on the independent dilatation or volume change $\tilde{J}$. The independent pressure p indicates the Lagrange multiplier enforcing the condition $\tilde{J} = \det\boldsymbol{F}$. $\Pi^{\mathrm{ext}}(\boldsymbol{\varphi})$ denotes the potential energy of external loads. Due to the principle of virtual work, Π_{STP} has to fulfill the condition

$$\delta\Pi_{\mathrm{STP}} \equiv \mathrm{D}_1\Pi_{\mathrm{STP}}(\boldsymbol{\varphi},\tilde{J},p)[\delta\boldsymbol{\varphi}] + \mathrm{D}_2\Pi_{\mathrm{STP}}(\boldsymbol{\varphi},\tilde{J},p)[\delta\tilde{J}] + \mathrm{D}_3\Pi_{\mathrm{STP}}(\boldsymbol{\varphi},\tilde{J},p)[\delta p] = 0 \tag{2}$$

where

$$\mathrm{D}_i f(x_1,\ldots,x_i,\ldots,x_n)[\delta x_i] := \left.\frac{\mathrm{d}}{\mathrm{d}\epsilon}\right|_{\epsilon=0} f(x_1,\ldots,x_i+\epsilon\,\delta x_i,\ldots,x_n) \tag{3}$$

denotes the functional derivative in direction of the variation δx_i. Moreover, we recall that the variations $\delta\boldsymbol{\varphi}$, $\delta\tilde{J}$ and δp are modifications at fixed time t. In Fig. 1, we illustrate these variations at two time points of a motion. We illustrate a virtual displacement $\delta\boldsymbol{\varphi}$ at time t_1, and a virtual volume change $\delta V_2 = (\partial V_2/\partial\tilde{J})\,\delta\tilde{J}$ due to a virtual pressure δp at a subsequent time point t_2. Applying Eq. (3) to Eq. (1), we obtain

$$\begin{aligned} 0 = &\int_{\mathscr{B}_0}\left(2\boldsymbol{F}\frac{\partial\,\Psi^{\mathrm{iso}}(\bar{\boldsymbol{C}})}{\partial\boldsymbol{C}} + p\,\mathrm{cof}\boldsymbol{F}\right) : \frac{\partial(\delta\boldsymbol{\varphi})}{\partial\boldsymbol{X}}\,\mathrm{d}V + \delta\Pi^{\mathrm{ext}}(\boldsymbol{\varphi}) - \int_{\mathscr{B}_0}\delta p\left[\tilde{J} - \det\boldsymbol{F}(\boldsymbol{\varphi})\right]\mathrm{d}V \\ &+ \int_{\mathscr{B}_0}\delta\tilde{J}\left[\frac{\partial\,\Psi^{\mathrm{vol}}(\tilde{J})}{\partial\tilde{J}} - p\right]\mathrm{d}V \end{aligned} \tag{4}$$

with the cofactor $\mathrm{cof}\boldsymbol{F} := \det\boldsymbol{F}\,\boldsymbol{F}^{-T}$. Consequently, Π_{STP} leads to the Euler–Lagrange equations

$$\tilde{J} = \det\boldsymbol{F}(\boldsymbol{\varphi}) \qquad p = \frac{\partial\,\Psi^{\mathrm{vol}}(\tilde{J})}{\partial\tilde{J}} \tag{5}$$

Using these principles in dynamics, we thus arrive at a differential algebraic equation system (see Ref. [5]).

As a consequence of this procedure, Eqs. (5) define the dilatation $\tilde{J}_{t_i}$ and the pressure p_{t_i} at time t_i of a motion directly and indirectly, respectively, *only* by the corresponding deformation mapping $\boldsymbol{\varphi}_{t_i}$. Therefore, the mixed variables $\tilde{J}$ and p are not *directly* related with continuous time curves $t \mapsto \tilde{J}(t,\mathscr{B}_0)$ and $t \mapsto p(t,\mathscr{B}_0)$, respectively. In contrast, any configuration $\boldsymbol{\varphi}_{t_i}(\mathscr{B}_0)$ is directly related with its previous configuration $\boldsymbol{\varphi}_{t_{i-1}}(\mathscr{B}_0)$, because the equation of motion includes the partial time derivative of the material velocity $\boldsymbol{v} := \partial\boldsymbol{\varphi}/\partial t$. Hence, we require initial positions and initial velocities for defining a unique motion $t \mapsto \boldsymbol{\varphi}(t,\mathscr{B}_0)$.

On the other hand, at least since Ref. [6], we know of the high importance of an existing time curve of a deformation measure in the framework of energy–momentum schemes. We recall that energy–momentum schemes are time integration algorithms, which preserve the time evolution of energy and momentum functions (see Ref. [7]). The Hu–Washizu procedure applied to the principle of virtual work or the Hamiltonian principle of least action, respectively, introduces a deformation measure as additional mixed field variable (see Refs. [8,9]). Since additionally introduced mixed variables define the energy functions, they therefore have to be evaluated at each time point t_i. Of course, this evaluation during time-stepping is very easy if for each mixed variable a direct time curve exists. But, it can be relatively complicated if we need the deformation mappings $\boldsymbol{\varphi}_{t_i}$ and $\boldsymbol{\varphi}_{t_{i-1}}$ (cp. Ref. [10]).

In the past, a time curve of a deformation measure is additionally introduced, and therefore called *assumed* strain approximation in time (see Refs. [11,12]). However, in Ref. [13], it is shown that the assumed strain approximation in time follows from a variational principle automatically, if the principle of virtual power is used. Therefore, in this paper, we combine the Hu–Washizu procedure with the *principle of virtual power*.