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Equivalent Legendre polynomials: numerical integration of discontinuous functions in the finite element methods

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Abstract The efficiency of numerical integration of discontinuous functions is a challenge for many different computational methods. To overcome this problem, we introduce an efficient and simple numerical method for the integration of discontinuous functions on an arbitrary domain. In the proposed method, the discontinuous function over the domain is replaced by a continuous equivalent function using Legendre polynomials. The method is applicable to all Ansatz spaces with continuous basis functions, and it allows to use standard numerical integration methods in the entire domain. This imposed continuous function is called equivalent Legendre polynomial (ELP). Several numerical examples serve to demonstrate the efficiency and accuracy of the proposed method. Based on $2D$ and $3D$ problems of linear elastostatics, the introduced method is further evaluated in the concept of the fictitious domain finite element methods.

Keywords Legendre polynomials · discontinuous functions · finite element methods · spacetrees.

1 Introduction

In the past few decades, different authors have introduced various novel finite element methods (FEMs) with the aim of simple mesh generation. Due to the simple mesh, it is almost impossible to avoid having discontinuities within elements – and therefore these methods face elements with discontinuous integrands. Such methods are, for example, the extended finite element method (XFEM) [7, 36] and the generalized finite element method (GFEM) [33, 34], or fictitious domain FEMs such as the Finite Cell Method (FCM) [29, 13]. In these methods, the element involves discontinuous functions such as the step function or the Heaviside function. Due to the high computational cost and the complexity of the related numerical integration, this is the major bottleneck of these methods. Therefore, the integration of discontinuous functions is of great interest.

The corresponding integral with a discontinuous function in the FEMs is of the form Eq. (1)

$$\int_{\Omega_e} \mathcal{D}(\mathbf{x})\mathcal{P}(\mathbf{x})\, d\Omega. \tag{1}$$

where Ω_e is the parent domain of element e, $\mathcal{P}(\mathbf{x})$ is a smooth function derived from the finite element shape functions, and $\mathcal{D}(\mathbf{x})$ is a discontinuous function. In the framework of the XFEM, $\mathcal{D}(\mathbf{x})$ defines the Heaviside function $\mathcal{H}(\mathbf{x})$ – and in the context of the FCM, it defines the step function $\mathcal{S}(\mathbf{x})$, which is one within the physical domain and zero in the fictitious domain.

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