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A fuel-based signal optimization model

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ABSTRACT

This paper considers fuel-based signal optimization based on a model composed of a description of the fuel consumption by idling, stopped vehicles, and of fuel consumption by vehicles accelerating from stop until they pass the stop line, and defines stochastic effects of vehicle movements which consume excess fuel. The fuel-based signal optimization model is examined and validated using simulations. The performance of the model is compared with other signal optimization models, including Webster's model, TRANSYT-7F, and Synchro. Various optimal signal settings are simulated through the simulation-assignment model, DynaTAIWAN.

1. Introduction

According to the *Energy Statistics Annual Report* of the Bureau of Energy (Ministry of Economic Affairs, 2011), Taiwan's oil consumption was 120.3 million of oil equivalent kiloliters in 2010, 13% of it was by transportation. Among modes, fuel consumption and emissions from road vehicles are the highest. Two approaches for addressing this involve improvements in vehicle engines and more efficient use of vehicles. The focus of this paper is on the latter, and in particular on the optimization of signal settings that embrace environmental perspectives.

2. Fuel-based signal optimization model

To optimize signal timing taking into account fuel use, the analytical fuel consumption model (AFCM) developed by Liao and Machemehl (1997) is extended and used in a fuel-based signal optimization derivation. The model is developed based on geometric configurations, vehicle movement statuses, and cycle phases. The original AFCM model is estimated for the inbound approach, intersection, and the outbound approach over the effective red time, time from green onset to $t0$, and time from $t0$ to the end of the green phase, where $t0$ indicates the time extension from the onset of green to the point that arrivals are equal to the discharge.

The simplified AFCM used is;

$$TF = \sum_{i=1}^{n}\left\{\frac{1}{2}q_i r_i\left(\frac{r_i}{C}\right)f_a + \frac{1}{2}q_i r_i\left(\frac{t0_i}{C}\right)f_b + \frac{x_i^2}{2(1-x_i)}f_c\right\} \tag{1}$$

where n is number of signal phases, C is cycle time (in seconds), q_i is critical lane flow of phase i, r_i is effective red time of phase i, $t0_i$ is time $t0$ of phase i, after the green time starts, at time $t0$ the arrivals are equal to the discharge, x_i is q_iC/g_is_i (degree of approach saturation), g_i is effective green time of phase i, s_i is saturation flow rate on approach i, f_a, f_b, and f_c are fuel consumption rates, and TF is average fuel consumption for critical lanes during one signal cycle.

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The model thus includes three elements: $1/2q_ir_i(r_i/C)f_a$, represents fuel consumption by stopped vehicles that have an idle fuel consumption rate; $1/2q_ir_i(t0_i/C)f_b$, is fuel consumption by vehicles accelerating from stop until they pass the stop line; and $x_i^2/2(1-x_i)f_c$, is the stochastic effects of vehicle movements that consume excess fuel. For a particular intersection, optimum conditions are obtained by minimizing the fuel consumption with respect to cycle time.

In deriving an expression for the optimum cycle time to minimize fuel consumption, the ratio the green phase time to green time ($g_i/\sum g_i$) should be the same as the corresponding ratio y_i to the sum of y_i ($y_i/\sum y_i$), where y_i is the maximum ratio of flow to saturation flow served by the green indicator (q_i/s_i). If we let $Y = \sum y_i$ and $k_i = y_i/Y$, we can describe the relationship between green time and flow as;

$$y_i = k_iY \tag{2}$$

$$g_i = k_iC - k_iL \tag{3}$$

$$r_i = C - k_iC - k_iL \tag{4}$$

$$q_i = k_is_iY \tag{5}$$

$$x_i = \frac{q_iC}{g_is_i} = y_i\frac{C}{g_i} = \frac{YC}{(C-L)} \tag{6}$$

where L is lost time for a cycle. Rearranging Eq. (1) we obtain;

$$TF = \sum_{i=1}^{n}\left\{\frac{1}{2}\left[k_is_iYf_a + k_is_iY\frac{k_iY}{1-k_iY}f_b\right]\left[\frac{(c-k_ic+k_iL)^2}{C}\right] + \frac{1}{2}\frac{C^2Y^2}{(C-L)(C-L-CY)}f_c\right\} \tag{7}$$

Differentiating with respect to the cycle time, C, yields $dTF/dC = 0$ in Eq. (8) for minimum fuel consumption.

$$\begin{aligned}\frac{dTF}{dC} = \sum_{i=1}^{n}&\left\{\frac{1}{2}\left[k_is_iYf_a + k_is_iY\frac{k_iY}{1-k_iY}f_b\right]\left[\frac{2(C-k_iC+k_iL)(1-k_i)}{C} - \frac{(C-k_iC+k_iL)^2}{C^2}\right]\right.\\ &\left.+\frac{1}{2}Y^2f_c\left[\frac{2C}{(C-L)(C-L-CY)} - \frac{C^2}{(C-L)^2(C-L-CY)} - \frac{C^2(1-Y)}{(C-L)(C-L-CY)^2}\right]\right\} = 0\end{aligned} \tag{8}$$

According to Webster's (1958) derivation, the optimal cycle length for delay minimization is approximately equal to twice of the minimum cycle, $2C_m$. A pre-selected optimum cycle length $2C_m$ is chosen to simplify the term $C-L-CY$ in Eq. (8) because the term does not have any major effect on the optimum cycle length for fuel consumption minimization. C_m is the shortest cycle that allows all incoming traffic to pass through the intersection in the same cycle. It is the sum of the lost time per cycle and the time necessary to pass all traffic through the intersection at the maximum possible rate, i.e.

$$C_m = L + \sum_{i=1}^{n} C_m\frac{q_i}{s_i} \tag{9}$$

where q_i/s_i is the greatest flow to saturation flow for the ith phase. Therefore,

$$C_m = L + C_m\sum_{i=1}^{n} y_i = L + C_mY = \frac{L}{1-Y} \tag{10}$$

Since the pre-selected optimum cycle C_o, is $2C_m = 2L/(1-Y)$, and L can be replaced by $C_o(1-Y)/2$ for the term '$C-L-CY$'; thus,

$$C - L - CY = C - \frac{C(1-Y)}{2} - CY = \frac{C(1-Y)}{2} \tag{11}$$

Eq. (8) can then be reduced to

$$\frac{dTF}{dC} = \sum_{i=1}^{n}\left\{\frac{1}{2}\left[k_is_iYf_a + k_is_iY\frac{k_iY}{1-k_iY}f_b\right]\left[\frac{2(C-k_iC+k_iL(1-k_i))}{C} - \frac{(C-k_iC+k_iL)^2}{C^2}\right] + \frac{1}{2}Y^2f_c\frac{-2C}{(C-L)^2(1-Y)}\right\} = 0 \tag{12}$$

Let

$$M = \frac{1}{2}(1-Y)\left[k_is_iYf_a + k_is_iY\frac{k_iY}{1-k_iY}f_b\right] \tag{13}$$

$$N = Y^2f_c \tag{14}$$

By multiplying $C^2(C-L)^2(1-Y)$ into this equation, Eq. (12) becomes

$$\frac{dTF}{dC} = \sum_{i=1}^{n}\left\{M[(1-k_i)^2C^4 - 2(1-k_i)^2LC^3 + (1-2k_i)L^2C^2 + 2k_i^2L^3C - k_i^2L^4\right] - NC^3 = 0 \tag{15}$$