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Design, optimization and realization of a ferroelectric liquid crystal based Mueller matrix ellipsometer using a genetic algorithm

Lars Martin S. Aas *, Daniel G. Skåre, Pål G. Ellingsen, Paul Anton Letnes, Morten Kildemo

Department of Physics, The Norwegian University of Science and Technology (NTNU), N-7491 Trondheim, Norway

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ABSTRACT

The design of complete broadband polarimeters with high performance is challenging due to the wavelength dependence of optical components. An efficient genetic algorithm computer code was recently developed in order to design and re-optimize complete broadband Stokes polarimeters and Mueller matrix ellipsometers. Our results are improvements of previous patented designs based on two and three ferroelectric liquid crystals (FLCs). FLC based polarimeters are suited for broadband hyperspectral imaging, or multichannel spectroscopy applications. We have realized and implemented one design using two FLCs and compare the spectral range and precision with previous designs.

1. Introduction

A polarimeter is an instrument that measures the polarization state of light to gain information about light sources, or materials interacting with polarized light. By measuring how the polarization of light is altered after being reflected from a smooth surface, the technique is often referred to as ellipsometry.

The need for fast broadband Mueller matrix ellipsometers and Stokes polarimeters result in challenging design problems when using active polarization modulators which are intrinsically strongly dispersive. Although designs based on the Fresnel rhomb and alike are nearly achromatic, these are well suited for neither imaging nor high speed applications. In the case of polarimeters and Mueller matrix ellipsometers based on liquid crystal modulators, the direct search space may become huge [1] and standard optimization methods can evidently result in local minima far away from the optimum. An efficient genetic algorithm (GA) computer code was recently developed in order to design and re-optimize complete broadband Stokes polarimeters and Mueller matrix ellipsometers (MME) [1]. This code is used here to search systems generating and analyzing optimally selected polarization states, in order to reduce the propagation of noise from the intensity measurements to the Mueller matrix elements. Although the GA code was initially motivated by the challenging task of searching the components, states and azimuthal orientations for optimally conditioned broadband liquid crystal based polarimeters [1,2], the software is written in a versatile manner in order to handle general polarimeters based on any polarization changing components. For small scale production, we propose that the GA can be used to re-optimize the design due to imperfect polarization components, e.g. due to small deviations in the specifications of the optical components. Any addition of "non-conventional" polarization altering components in the polarimeter, such as mirrors and prisms can be included in the GA, given that the dispersive properties of such components are known.

A classical GA [3,4] was chosen to optimize the polarimeters based on ferroelectric liquid crystals (FLCs). FLC based polarimeters were first proposed by Gandorfer [5], and Jensen and Peterson [6]. They have the advantage of being fast [7] and having no moving parts, which is an advantage for imaging applications. A commercial FLC multichannel spectroscopic (430–850 nm) Mueller matrix ellipsometer [16] is available from Horiba Yvon Jobin. The FLC system is based on optical components with known properties [8,7]. Its overall performance depends on the components in a complex manner. Traditional optimization methods are hampered by local minima in the large search space. A genetic optimization algorithm is more robust and will normally move out of local minima resulting in a polarimeter design with less noise amplification on a broader spectral range.

In this work, a new design for the commercial FLC based MM16 system has been implemented. Furthermore, we demonstrate how the GA may be used in small scale production, where we may simply re-optimize the design in the case of an off-specification component.

2. Theory

The complete polarization state of light, including partially polarized states, can be expressed concisely using the Stokes vector.

* Corresponding author.
E-mail address: lars.martin.aas@gmail.com (L.M.S. Aas).

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It completely describes the polarization state with four real elements [9]

$$\mathbf{S}=\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}=\begin{bmatrix} \left\langle E_{0,x}(t)^2\right\rangle+\left\langle E_{0,y}(t)^2\right\rangle \\ \left\langle E_{0,x}(t)^2\right\rangle-\left\langle E_{0,y}(t)^2\right\rangle \\ 2\left\langle E_{0,x}(t)E_{0,y}(t)cos\delta(t)\right\rangle \\ 2\left\langle E_{0,x}(t)E_{0,y}(t)sin\delta(t)\right\rangle \end{bmatrix},$$

where $\langle \cdots \rangle$ denotes time average over, in general, quadratic time dependent orthogonal electric field components $(E_{0,x}(t))$ and $(E_{0,y}(t))$ and phase $(\delta(t))$.

The change of a polarization state can be described by a 4×4 real-valued transformation matrix called a Mueller matrix, **M**, connecting an incoming Stokes vector $\mathbf{S}_{in}$ to an outgoing Stokes vector

$$\mathbf{S}_{out}=\mathbf{M}\mathbf{S}_{in}. \tag{1}$$

Any linear interaction of light can be described by the Mueller matrix. A Mueller matrix can describe a range of polarization effect, such as diattenuation (different amplitude transmittance or reflectance for different polarization modes), retardance (i.e. changing δ), and depolarization (which increases the random component of the electric field).

A Stokes polarimeter consists of a polarization state analyzer (PSA) capable of determining the Stokes vector by performing at least four intensity measurements. For a given state (i), the polarization altering properties of the PSA can be described by its Mueller matrix $\mathbf{M}^{PSA}(i)$, which can be found as the matrix product of the Mueller matrices of all the optical components in the PSA. These components are a linear polarizer, and a number of phase retarders (e.g. FLCs and waveplates), see Fig. 1. An FLC is a phase retarder which can be electronically switched between two states. The difference between the states corresponds ideally to a rotation of the fast axis by 45° ($\theta(0)=\theta_0$ and $\theta(1)=\theta_0+45$ °). By using a linear polarizer and two FLCs as a PSA, one can generate $2^2=4$ different projection states and by using three FLCs one can generate $2^3=8$ states, etc.

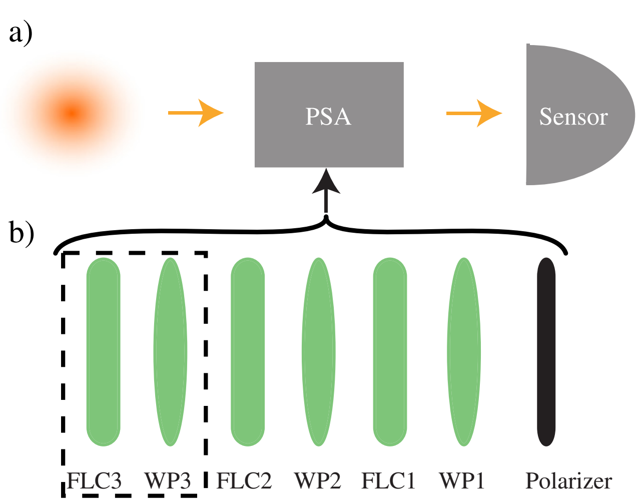

Fig. 1. A schematic drawing of a polarimeter, (a) shows a general polarimeter where the polarization state of incident light is analyzed by the polarization state analyzer and a light intensity detector. In (b) the components of a polarization state analyzer is exemplified through a combination of two or three FLCs and waveplates (WPs) and a linear polarizer.

If an unknown polarization state with Stokes vector **S** passes through the PSA, for a state *i*, the detector will measure an intensity *I* depending only on the first row of $\mathbf{M}^{PSA}(i)$

$$I=\sum_{j=1}^{4}\mathbf{M}_{1,j}^{PSA}(i)\mathbf{S}_j.$$

The intensity can be considered to be the projection of **S** along a Stokes vector equal to $\mathbf{M}_{1,1..4}^{PSAT}$, where T denotes the transpose. These Stokes vectors are organized as rows in the system matrix **A**. When operating on a Stokes vector the result is

$$\mathbf{b}=\mathbf{A}\mathbf{S}.$$

Here **b** is a vector composed by the intensity measurements at the different projection states. An unknown Stokes vector can then be found by $\mathbf{S}=\mathbf{A}^{-1}\mathbf{b}$. The noise in **S** comes from the measurement noise in **b** but is amplified by the condition number (κ) of **A** [10]. Therefore κ of a polarimeter should be as small as possible [11]. A low κ indicates that the probing polarization states are close to orthogonal.

The condition number of **A** is given as $\kappa=\|\mathbf{A}\|\|\mathbf{A}^{-1}\|$, which for the 2-norm is equal to the ratio of the largest to the smallest singular value of the matrix [10]. Theoretically the best condition number that can be achieved for a polarimeter is $\kappa=\sqrt{3}$ [11]. If four optimal states can be achieved, no advantage is found by doing a larger number of measurements with different states, compared to repeated measurements with the four optimal states [12]. If, however, these optimal states cannot be produced ($\kappa>\sqrt{3}$), the condition number, and hence the error, can be reduced by measuring more than four states. For an FLC based polarimeter this is accomplished by using three FLCs in the PSA, with up to three waveplates (WP) coupled to the FLCs to reduce the condition number (see Fig. 1), or components with more than two states, such as liquid crystal variable retarders (LCVR). In this case **A** will not be a square matrix, and the *Moore–Penrose pseudoinverse* is then used to invert **A** [2].

To measure the Mueller matrix of a sample, it is necessary to illuminate the sample with at least four different polarization states. The Stokes vectors of these states can be organized as columns in a polarization state generator (PSG) system matrix **W**. After interaction with the sample the product **MW** gives the resulting four Stokes vectors. They are then measured by the PSA, yielding the intensity matrix $\mathbf{B}=\mathbf{AMW}$. The Mueller matrix can then be found by multiplying the expression by $\mathbf{A}^{-1}$ and $\mathbf{W}^{-1}$ from each side, $\mathbf{M}=\mathbf{A}^{-1}\mathbf{B}\mathbf{W}^{-1}$. The PSG may be constructed from the same optical components as the PSA.

3. Fitness evaluation

It has already been established that κ should be as small as possible in order to reduce noise in the polarimetric measurements. It is a fairly trivial exercise to optimize κ for a single wavelength. However, there are two sources of wavelength dependence of the optical properties of the components.

One of these is the explicit wavelength dependence of the retardance Δ_R, which can be calculated as [13]

$$\Delta_R=\frac{2\pi l(\Delta n)}{\lambda_0}, \tag{2}$$

where l is the physical thickness of the component (e.g. waveplate or FLC), λ_0 is the vacuum wavelength of the light, and Δn is the birefringence of the material. Birefringence is the difference in refractive index between the fast axis (index of refraction n_f) and the slow axis (n_s), i.e. $\Delta n=|n_f-n_s|$ [13]. There is an explicit wavelength dependence in Eq. (2), which complicates the design of the PSA. A weaker, but still important, effect is the wavelength dependence of the birefringence,

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