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Dynamic strength around two interacting piezoelectric nano-fibers with surfaces/interfaces in solid under electro-elastic waves

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ABSTRACT

Considerable efforts have been devoted to the characterization of the strength at the surfaces/interfaces of piezoelectric nanocomposites. In this paper, the multiple scattering of electro-elastic waves from two piezoelectric nano-fibers in piezoelectric matrix is considered and the dynamic stress at the interface is obtained. A decoupling function is introduced to solve the governing equation in piezoelectric materials. The displacement and electric potential are described by a wave function expansion method. The addition theorem for cylindrical wave function is used to obtain the total coupling wave field. The conventional surface/interface model of Gurtin and Murdoch is extended to the case of coupling stress and electric displacement. The dynamic stress concentration factor around two nano-fibers is obtained. Through numerical analysis, it is found that the interaction between the two nano-fibers has a significant effect on the coupling of stress and electric displacement at the interface, especially in the region of high frequency. The interacting effect of the two piezoelectric interfaces is also related to the properties of the nano-fibers and interface. To show the validity of the dynamic model, comparison with existing results is also given.

1. Introduction

Nanocomposites exhibit ultra-high yield and fracture strengths, superior wear resistance, and enhanced super-plastic formability. With the demand for miniaturization of electron device and rapid development of nanotechnology, nanotechnology is making profound progress in piezoelectric composites [1,2]. Recently, intensive efforts have been devoted to synthesize and characterize the response of piezoelectric nanocomposites under many kinds of loadings.

In recent years, interest in the wave propagation problem involving nanocomposites has increased appreciably because of its important applications. These waves include compressional waves [3–5], shear waves [6,7], and other harmonic waves [8]. Through analyzing the propagation and scattering of waves in nanocomposites, the dynamic response of nano structures can be predicted precisely.

In nanocomposites, the surface or interface effects are particularly important due to a large ratio of surface area to volume. Surface or interface can be interpreted as an elastic membrane, or an interfacial thin layer, between the inclusions and the matrix. In the past years, the surface/interface model of Gurtin and Murdoch [9] is widely used to analyze the wave scattering in nanocomposites. In this model, however, the electric displacement effect on the stress at the surface/interface is not considered. In piezoelectric surface/interface, the coupling of stress and electric displacement in the vicinity of surfaces as well as the interplay between electricity and elasticity therein are more complicated.

In practical engineering, the electron device made from piezoelectric nanocomposites may suffer from many kinds of loading and the stress concentration around the discontinuities will degrade their function in their serving life. Recently, Huang and Yu [10] studied the electromechanical behavior of a piezoelectric ring, and the influence of surface piezoelectricity on the strength is considered. Michalski et al. [11] proposed a continuum theory to simulate the piezoelectric response of nanotubes and nanowires. Delobelle et al. [12] studied the strength of [Pb(Zr,Ti)O_3] and [Pb($Mg_{1/3}Nb_{2/3}$)$_{1-x}$$Ti_x$$O_3$] sputtered thin films. Chen [13] studied the macroscopic mechanical behavior of two-phase fibrous piezoelectric composites, and the effects of surface stress and surface electric displacement were both considered. In piezoelectric composites, the electro-elastic waves can simulate many high frequency loadings, and have lots of engineering application; however, no investigations on this problem are available now. Most recently, only the scattering of electro-elastic waves at the surface/interface of a nano-fiber in piezoelectric composites was investigated [14].

In this paper, the work of Fang and Liu [14] is extended to the two interacting piezoelectric nano-fibers in piezoelectric composites, and the multiple scattering of electro-elastic waves at the surfaces/interfaces of the two nano-fibers is studied. The interacting effects of the two surfaces/interfaces on the dynamic stress and electric displacement around the nano-fibers are considered. The displacement and electric potential in piezoelectric composites are expressed by wave function expansion. The conventional surface/interface model is extended to

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the piezoelectric surface/interface. The numerical solutions of dynamic stress concentration factor around the two nano-fibers are graphically illustrated. The effects of the distance between the two nano-fibers, the coupling of stress and electric displacement, the frequency of electro-elastic waves, and the material properties of nano-fibers and surfaces/interfaces on the dynamic stress in the matrix material are analyzed. This modeling can be extended to characterize the mechanical stability of ultra-thin layers of coating onto the array of nanowires structures under impacts.

2. Problem formulation and governing equation

It is supposed that two piezoelectric nano-fibers with surface/interfaces are located within a large region. The geometry is depicted in Fig. 1. For convenience, three coordinate systems are introduced. Two local coordinate systems are located in the centers of the two nano-fibers, and another is the global coordinate system. It is assumed that the piezoelectric nano-fibers are poled in the z-direction exhibiting transversely isotropic behavior (hexagonal symmetry). For convenience of notation, the matrix is denoted by 'M', and the two nano-fibers, are denoted by 'F'.

An anti-plane wave with frequency ω propagating in the positive x-direction is considered, as shown in Fig. 1. For the anti-plane dynamic problem of piezoelectric composites, the coupling governing equation is expressed as

$$c_{44}\nabla^2 u + e_{15}\nabla^2\phi = \rho\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$e_{15}\nabla^2 u - \varepsilon_{11}\nabla^2\phi = 0 \tag{2}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator in the variables x and y. u and $\phi(j = x, y)$ are anti-plane displacement and in-plane electric displacement, respectively; c_{44} is the elastic stiffness of piezoelectric materials measured in a constant electric field, ε_{11} is the dielectric constant measured in a constant strain, and e_{15} is the piezoelectric constant. The elastic stiffness, dielectric constant, and piezoelectric constant of the nano-fiber is denoted as c_{44}^F, ε_{11}^F, and e_{15}^F, and those of the matrix is c_{44}^M, ε_{11}^M, and e_{15}^M.

Assume that another electro-elastic field ψ is expressed as

$$\psi = \phi - \lambda u, \tag{3}$$

where $\lambda = e_{15}/\varepsilon_{11}$.

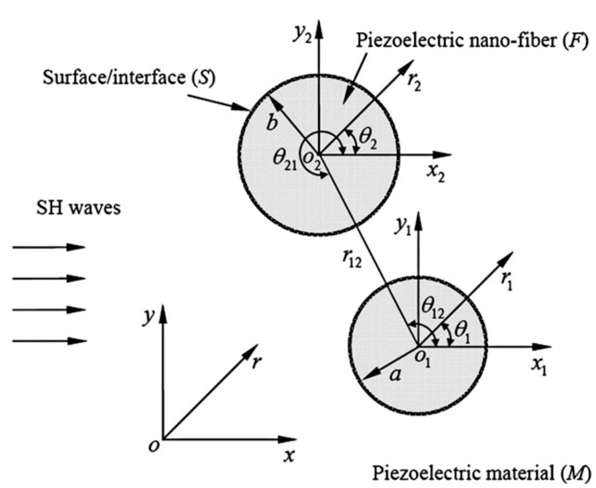

Fig. 1. Schematic diagram of two piezoelectric nano-fibers in piezoelectric composites and incident wave.

From Eqs. (1)–(3), the following equations can be obtained

$$\nabla^2 u = \frac{1}{c_{SH}^2}\frac{\partial^2 u}{\partial t^2}, \tag{4}$$

$$\nabla^2\psi = 0, \tag{5}$$

where $c_{SH} = \sqrt{\mu^e/\rho}$ with $\mu^e = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}$ being the wave speed of electro-elastic waves in the matrix. It should be noted that Eqs. (4) and (5) are the classical wave equation, and can be solved by wave function expansion method.

In the piezoelectric bulk, considering the anti-plane character, the mechanically and electrically coupled constitutive equations can be written as

$$\tau_{xz} = c_{44}\frac{\partial u}{\partial x} + e_{15}\frac{\partial\phi}{\partial x}, \tau_{yz} = c_{44}\frac{\partial u}{\partial y} + e_{15}\frac{\partial\phi}{\partial y}, \tag{6}$$

$$D_x = e_{15}\frac{\partial u}{\partial x} - \varepsilon_{11}\frac{\partial\phi}{\partial x}, D_y = e_{15}\frac{\partial u}{\partial y} - \varepsilon_{11}\frac{\partial\phi}{\partial y}, \tag{7}$$

where τ_{jz} and D_j $(j = x, y)$ are the shear stress and electric potential, respectively.

3. Computational models of surfaces/interface around the two nano-fibers considering piezoelectric effect

Around the two nano-fibers, the surface/interface usually penetrates several atomic layers into the bulk. The material surface is usually modeled as a medium with material properties different from those of the bulk. The electrical and mechanical properties in the vicinity of surfaces as well as the interplay between the electricity and elasticity therein are more complicated.

In conventional nanocomposites, the surface/interface elasticity theory of Gurlin and Murdoch is widely used [9]. It should be noted that only the surface stress exists in this model. In this study, this model is extended to the piezoelectric nanocomposites. In surface/interface, the electrical and mechanical properties in the vicinity of surfaces as well as the interplay between electricity and elasticity therein are more complicated. The equilibrium and constitutive equations in solids are the same as those in the classical theory of coupling electricity–elasticity, but the presence of surface stress gives rise to a nonclassical boundary condition.

The surface/interface stress $\sigma_{\alpha\beta}^S$ and surface electric displacement D_α^S in solids are linked with the surface strain tensor $\varepsilon_{\alpha\beta}^S$ and surface electric field E_α^S by the relations

$$\sigma_{\alpha\beta}^S = \tau_0\delta_{\alpha\beta} + \frac{\partial\Gamma\left(\varepsilon_{\alpha\beta}^S, E_\alpha^S\right)}{\partial\varepsilon_{\alpha\beta}^S}, D_\alpha^S = D_{\alpha 0} + \frac{\partial\Gamma\left(\varepsilon_{\alpha\beta}^S, E_\alpha^S\right)}{\partial E_\alpha^S}, \alpha,\beta = x,y,z, \tag{8}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta function and the surface energy density Γ depends on the in-plane strain at the surface as well as on the surface electric field. τ_0 is the residual surface stress. $D_{\alpha 0}$ is the residual surface electric displacement.

In this study, only the anti-plane problem is considered, and the surfaces/interfaces of the two nano-fibers process the same material behavior. In the coordinate systems (r_p, θ_p) positioned at the centers of two fibers, the constitutive relations can be expressed as

$$\sigma_{\theta zp}^S = c_{44}^S\varepsilon_{\theta zp}^S + e_{15}^S E_{\theta p}^S, p = 1,2, \tag{9}$$

$$D_{\theta p}^S = D_{\theta 0p} + e_{15}^S\varepsilon_{\theta zp}^S - \varepsilon_{11}^S E_{\theta p}^S, p = 1,2 \tag{10}$$

where c_{44}^S, ε_{11}^S, and e_{15}^S are the elastic stiffness, dielectric constant and piezoelectric constant of surface/interface. Determination of c_{44}^S, ε_{11}^S,