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Critical depth of cut and asymptotic spindle speed for chatter in micro milling with process damping

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ABSTRACT

This paper presents formulas for determining the critical depths of cut and asymptotic spindle speed for stable micro milling processes with process damping. The asymptotic spindle speed of a theoretically infinite stable depth of cut is shown to be proportional to the modal natural frequency, radial ploughing constant and radial immersion angle, but inversely proportional to the shearing related cutting constants and tool diameter. These formulas enable identifying the asymptotic speed, absolute stability limit, and in-process radial ploughing constant from experimental stability limits without requiring modal parameters. The presented model is validated by comparison with prior works and verified by experiments.

1. Introduction

The productivity of micro milling processes is often compromised by its low process stability because of low tool stiffness and high specific cutting force. Process damping associated with edge ploughing and flank interference, by contrast, improves machining stability [1], and can offer a viable means to improve the stability margin for micro milling.

General milling stability problems have been under extensive investigations and significant amount of work is still actively devoted to studying micro milling stability, where the intricacy of the edge geometry, size effect in the cutting and ploughing mechanism, and particularly, the identification and modeling of process damping and their effects on stability have been the foci of these investigations. In these investigations, frequency domain approaches can be used for their efficiency in constructing stability diagrams [2–4] whereas time domain simulation allows easy accommodation of the process nonlinearities and prediction of detailed process information such as the tool trajectory, surface texture, and chatter frequency [5–7]. The effects of edge preparation on micro milling stability has been analyzed through experimental investigation in [8].

The modeling and identification of process damping [9] and its effect on milling stability have been documented for general milling [10–14]. Stability analyses incorporating a process damping model in micro milling have been reported in [3,4]. The process damping coefficient in [3] was obtained from the ploughing force coefficient acquired from experimentally measured cutting forces, while finite element simulation was used to predict the process damping coefficients in [4] through a contact mechanics model.

Stability diagrams with process damping are generally characterized by an increasing stability trend at low speeds, forming a curved-up envelope of the critical limits toward the low-speed region. The critical stability limit for a turning process with process damping has been reported in [12,14]. But, the critical stability limits with respect to the cutting speed and the attributes of their envelope for a milling process have not been quantitatively analyzed.

By adopting the mechanistic local milling force model with process damping in [9], this paper extends the existing critical depth model for a symmetric structure without process damping in [15] to reveal the roles of process damping and other system parameters in characterizing the curve of critical stability limits. In the following, the equivalent process damping ratio and decoupled system characteristic equation are first derived. The formulas for the critical depth of cut and asymptotic speed are then presented, followed by model validation through comparison with published results and experiments.

2. Characteristic equation with process damping ratio

A micro milling process with single-mode symmetric dynamics is shown in Fig. 1(a), where the local cutting forces resulting from the shearing and ploughing effects depicted in Fig. 1(b) can be expressed as [9],

$$\begin{pmatrix} df_t \\ df_r \end{pmatrix} = \left\{ \begin{pmatrix} K_{ts} \\ K_{rs} \end{pmatrix} (\sin\theta \quad \cos\theta) \begin{pmatrix} \delta_{xd} \\ \delta_{yd} \end{pmatrix} \right\} db + \left\{ \begin{pmatrix} K_{tp} \\ K_{rp} \end{pmatrix} (\sin\theta \quad \cos\theta) \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} \right\} \frac{db}{v_c} \quad (1)$$

where the first curly bracket on the right represents the specific regenerative cutting force resulting from the dynamic feeds, $\{\delta_{xd}, \delta_{yd}\}^T = \{x(t) - x(t-T), y(t) - y(t-T)\}^T$ and T is the tooth period. K_{ts} and K_{rs} are the tangential and radial constants, respectively, for the

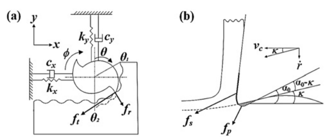

Fig. 1. (a) A schematic of micro milling process, (b) the local dynamic chip shearing and edge ploughing forces with tool penetration angle, κ.

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shearing mechanism. The second curly bracket accounts for the process damping forces caused by tool vibration in the radial direction. K_{tp} and K_{rp} are respectively the tangential and radial dynamic ploughing constants representing the specific ploughing force generated per radian change in the tool penetration angle, $k = \dot{r}/v_c$, where $\dot{r} = \{\sin\theta, \cos\theta\}(\dot{x}, \dot{y})^{\mathrm{T}}$ is the radial velocity and v_c is the cutting velocity. Transforming the local cutting forces into x–y directions and integrating them along the active cutting edges of the rotating cutter as in [16] enables a total dynamic force vector to be obtained in the form of

$$\begin{pmatrix} f_{xd}(t) \\ f_{yd}(t) \end{pmatrix} = \mathbf{K}_s(t)\begin{pmatrix} \delta_{xd}(t) \\ \delta_{yd}(t) \end{pmatrix} + \mathbf{C}_p(t)\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} \quad (2)$$

where $\mathbf{K}_s$ is the process stiffness matrix for the dynamic feed vector and $\mathbf{C}_p$ is the viscous processing damping coefficient matrix. These two matrices are periodic functions, but only the zero-order terms of the matrices are considered in this paper. The zero-order or average process stiffness matrix, $\mathbf{K}_{s0}$, can be expressed as [15]

$$\mathbf{K}_{s0} = \frac{NK_{ts}a}{2\pi}\mathbf{P}_s \quad (3)$$

where N is the number of flutes and a is the axial depth of cut. $\mathbf{P}_s$ is the zero-order directional matrix:

$$\mathbf{P}_s = \begin{bmatrix} 1 & -k_{rs} \\ k_{rs} & 1 \end{bmatrix}\begin{bmatrix} -0.25\cos 2\theta & 0.5\theta + 0.25\sin 2\theta \\ -0.5\theta + 0.25\sin 2\theta & 0.25\cos 2\theta \end{bmatrix}_{\theta_1}^{\theta_2} \quad (4)$$

where $k_{rs} = K_{rs}/K_{ts}$ is the radial force ratio and θ_1 and θ_2 are the entry and exit angles, respectively. Similarly, the average viscous process damping coefficient can be expressed as

$$\mathbf{C}_{p0} = \frac{NK_{tp}a}{2\pi v_c}\mathbf{P}_p \quad (5)$$

where $\mathbf{P}_p$, the zero-order directional matrix for the process damping, is the same as that for the regenerative forces except the radial force ratio in Eq. (4) is replaced by $k_{rp} = K_{rp}/K_{tp}$.

For a micro milling process, the spindle-holder-tool assembly can be approximated as a second-order isotropic dynamic system. Using the zero-order force models, the dynamic equation for the micro end mill with process damping can be obtained as

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} + \left\{\begin{bmatrix} c_s & 0 \\ 0 & c_s \end{bmatrix} + C_{p0}\right\}\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\mathbf{K}_{s0}\begin{pmatrix} \delta_{xd} \\ \delta_{yd} \end{pmatrix} \quad (6)$$

where m, c, and k arethe dominant modal parameters for the end mill. This equation can be solved for stability limits in the frequency domain by using methods presented in [3,4]. Because the axial depth appears in both the $\mathbf{K}_{s0}$ and $\mathbf{C}_{p0}$ matrix, the limiting stable depth has to be solved iteratively.

The coupled directional matrices in Eq. (4) can be decoupled in their eigenvalues, λ_i, and eigenvectors, $\mathbf{Q}_i$, as follows:

$$\mathbf{P}_s = \mathbf{Q}_s\begin{bmatrix} \lambda_s & 0 \\ 0 & \lambda_{s2} \end{bmatrix}\mathbf{Q}_s^{-1} \quad \text{and} \quad \mathbf{P}_p = \mathbf{Q}_p\begin{bmatrix} \lambda_p & 0 \\ 0 & \lambda_{p2} \end{bmatrix}\mathbf{Q}_p^{-1} \quad (7)$$

$\mathbf{P}_s$ and $\mathbf{P}_p$ have similar structures and thus similar eigenvalues and eigenvectors. The eigenvalues of $\mathbf{P}_s$ were expressed as [15]

$$\lambda_{i,i2} = \frac{(k_{ri}\theta_r \pm \sqrt{\delta})}{2}; \quad \delta = (1 + k_{ri}^2)\sin^2\theta_r - \theta_r^2 \quad (8)$$

For a positive discriminant, δ, of a lower radial immersion angle, $\theta_r = \theta_2 - \theta_1$, the two eigenvalues are real with the higher one, λ_i, being dominant in the stability sense, whereas the dominant complex eigenvalue of a higher immersion angle with a negative discriminant has a negative phase:

$$\lambda_i = |\lambda_i|e^{j\phi_{\lambda i}}; \quad |\lambda_i| = \frac{\sqrt{(k_{ri}\theta_r)^2 - \delta}}{2}; \quad \phi_{\lambda i} = \tan^{-1}\left(-\frac{\sqrt{-\delta}}{(k_{ri}\theta_r)}\right) \quad (9)$$

The subscript i in Eqs. (8) and (9) is substituted by s and p for the shearing and process damping effect, respectively.

Substituting the expressions for $\mathbf{P}_s$ and $\mathbf{P}_p$ in Eq. (7) into Eqs. (3) and (5), respectively, and through modal transformation of $\{x, y\}^{\mathrm{T}} = \mathbf{Q}_s\{q_1, q_2\}^{\mathrm{T}}$, Eq. (6) can be decoupled as

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \left\{\begin{bmatrix} c_s & 0 \\ 0 & c_s \end{bmatrix} + \begin{bmatrix} c_p & 0 \\ 0 & c_{p2} \end{bmatrix}\right\}\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_1 \end{bmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$
$$= \frac{-NK_{ts}a}{2\pi}\begin{bmatrix} \lambda_s & 0 \\ 0 & \lambda_{s2} \end{bmatrix}\begin{pmatrix} q_1 - q_1(t-T) \\ q_2 - q_2(t-T) \end{pmatrix} \quad \text{with}\, c_p = \frac{NK_{tp}a}{2\pi v_c}\lambda_p \quad (10)$$

if $\mathbf{Q}_s$ and $\mathbf{Q}_p$ are the same so that $\mathbf{Q}_s^{-1}\mathbf{Q}_p = 1$. q_1 and q_2 are the modal coordinates with q_1 being the dominant one with the less stable eigenvalue. Eigenvectors are generally different if their respective radial force ratios are different. However, this difference can be shown to diminish with a higher radial immersion, where both eigenvalue are complex and the approximation of $\mathbf{Q}_s^{-1}\mathbf{Q}_p = 1$ can be justified. For the case of slot milling, $\mathbf{Q}_s$ and $\mathbf{Q}_p$ are the same regardless of their radial constants. For $k_{rp} = 1$, 1.2, 1.5 and 2, the minimum immersion ratios are 0.41, 0.5, 0.6 and 0.72, respectively, to yield complex eigenvalues. Thus the radial immersion ratio is assumed to be higher than 0.6 for a k_{rp} of lower than 1.5 in this paper to justify this approximation. Because the cutting force ratio k_{rs} is generally lower than k_{rp}, both pairs of eigenvalues are complex values.c_p and c_{p2} in Eq. (10) are the average process damping coefficients for modes 1 and 2, respectively, and because q_1 is the dominant mode, only c_p needs to be considered. It can be seen from Eq. (10) that, for a complex λ_p, only the real part generates the viscous damping force. The imaginary part has the effect of strengthening the system stiffness, but it can be shown to generally exert a markedly less effect compared with the structural stiffness; therefore, it is not considered in this paper. The real part of λ_p can be found from Eq. (8) to be $k_{rp}\theta_r/2$ and the average process damping coefficient c_p and damping ratio ζ_p can then be written as

$$c_p = N\left(\frac{K_{rp}\theta_r}{4\pi}\right)\frac{a}{v_c} \,\text{and}\, \zeta_p = \frac{c_p}{2\sqrt{mk}} = \zeta_p' a; \quad \zeta_p' = \frac{NK_{rp}\theta_r}{8\pi\sqrt{mk}v_c} \quad (11)$$

where ζ_p' is the specific process damping ratio. These three damping related factors are shown to be proportional to the flute number, radial ploughing constant and radial immersion angle.

Based on the Laplace transform of the decoupled Eq. (10), the dominant system characteristic equation has the expression of

$$1 + \frac{NK_{ts}a}{2\pi}(1 - e^{-sT})\lambda_s H_p(s) = 0 \quad (12)$$

where the modal dynamics H_p has a total damping ratio of

$$\zeta_t = \zeta_s + \zeta_p; \quad \zeta_s = \frac{c_s}{(2\sqrt{mk})} \quad (13)$$

3. Critical depth of cut and asymptotic speed

For the characteristic equation in the form of Eq. (12), the critical depth of cut with only structural damping can be expressed as [15]

$$a_{c0} = \frac{2\pi k\zeta_s}{NK_{ts}}\frac{(1 + c_0{}^2)}{|\lambda_s|(c_0\cos\phi_{\lambda s} - \sin\phi_{\lambda s})} \quad \text{with}\, c_0 = 1 + \frac{2\phi_{\lambda s}}{\pi} \quad (14)$$

or in an alternative form of

$$a_{c0} = Ak\zeta_s \quad \text{with}\, A = \frac{4\pi}{NK_{ts}g_{rs}} \,\text{and}\, g_{rs}$$
$$= \frac{2|\lambda_s|(c_0\cos\phi_{\lambda s} - \sin\phi_{\lambda s})}{(1 + c_0{}^2)} \quad (15)$$

where g_{rs} is the radial immersion gain for the shearing mechanism. By substituting ζ_s with the total damping ratio into Eq. (15), the new critical stability limit with process damping becomes

$$a_c = \frac{Ak\zeta_s}{1 - Ak\zeta_p'}; \quad Ak\zeta_p' = \left(\frac{K_{rp}\theta_r}{K_{ts}g_{rs}}\right)\frac{f_n}{D\Omega/60} \quad (16)$$

with f_n in Hz and Ω in rpm. As the spindle speed decreases, thecritical stability depth increases and the envelope of the stability lobes curves upward. Two extreme conditions establish the limits of the critical curve. The lower limit of the curve is defined by a_{c0}, the absolute stability limit in high-speed regions without process damping, whereas the left margin is defined by the asymptotic speed, a limiting speed corresponding to an infinite stable depth.

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