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Chip geometry and cutting forces in gear shaping

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ABSTRACT

Shaping is a versatile process for machining gear teeth on complex monolithic components, which cannot otherwise be produced using other methods (like hobbing) due to geometric constraints. This paper presents a new model to accurately predict the chip geometry and cutting forces in shaping. Kinematics of gear shaping is modeled and verified. Cutter–workpiece engagement is predicted using a dexel-based geometric modeler, and refined by alpha-shape reconstruction. Varying rake and oblique angles are resolved along the cutting edges and used in three-dimensional force predictions. Simulated forces are shown to agree closely with those measured on a gear shaping machine.

1. Introduction

Gear shaping is the most widely used method for pre-machining internal gears [1]. It is especially suited for cases when tool maneuvering space is restricted. To achieve maximum load capacity and reduce running noise, pre-machined gears undergo heat treatment followed by hard finishing, such as grinding or honing [2]. Chip formation in gear shaping is very complex, despite the fact that the cutting speed and feedrates are typically kept constant. This is due to varying cutter–workpiece engagement (CWE) conditions arising from the gear generative motion, and prior cutting history. The toothed profile of the cutter can also result in multi-flank chips [3]. These complexities make the application of simple approaches, commonly used to model chip formation in processes like turning and milling, impractical.

As an alternative, geometric modeling kernels can be used to accurately resolve the CWE. Such kernels can be categorized into two kinds: solid (exact) and discrete (approximate). Bouzakis et al. [4] used a solid kernel (Solidworks) to model chip geometry in hobbing. Although solid modeling provides the maximum accuracy, due to its analytical approach, it is computationally intensive. In contrast, discrete representation is fast and robust. For these reasons, the latter has been widely adopted in collision detection and in-process workpiece modeling. In recent years, a number of studies have extended the use of discrete modelers to CWE calculation [5,6], including the modeling of chip geometry in bevel gear cutting [7]. In this paper, a discrete modeling method based on multi-dexel volume representation [8] is used to extract CWE at each time step during gear shaping. This data is refined to estimate the chip geometry and simulate cutting forces.

In the remainder of this paper, the kinematic modeling of the gear shaping operation, CWE estimation, and proposed force prediction algorithm are presented in Sections 2–4. Experimental validation is presented in Section 5, followed by the conclusions.

2. Kinematics of gear shaping

While this paper studies spur gear shaping, many of the concepts discussed are also applicable to helical gear shaping, which considered as the continuation of this study. The principle movements in spur gear shaping are illustrated in Fig. 1a and b. The cutting motion is delivered by a reciprocating shaper cutter. To obtain a true generating process, a rotary feed motion is used, which emulates the rolling of the pitch circles. Different infeed strategies can be used to set the radial depth of cut. The radial feed

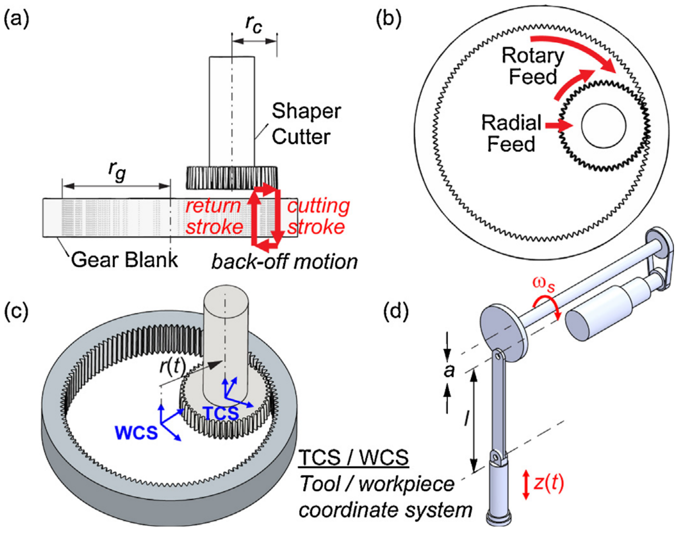

Fig. 1. Principle movements and geometries in gear shaping: (a) Cutting, (b) feed movements, (c) shaping of an internal gear, (d) slider-crank.

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is disabled when the nominal center-to-center distance is reached. To avoid collision between the shaper cutter and the in-process workpiece, a radial back-off motion is executed at the end of each cutting stroke, which keeps the tool away from the workpiece during the return stroke. This motion does not affect the position of the tool while cutting, and is thus not modeled.

The cutting edges at the bottom face of the cutter have the shape of a symmetric involute gear tooth projected on a cone of constant slope. The flanks of each tooth are generated by two involutes from a common base circle, and are spaced so that tooth thickness at the pitch circle is known. The tip of the tooth is a circular arc. It is convenient to represent cutting edges in the Tool Coordinate System (TCS, shown in Fig. 1c). To resolve the uncut chip geometry, the trajectory of the cutting edges also has to be calculated in the Workpiece Coordinate System (WCS), which at any time t, can be accomplished using the homogenous transform:

$$M_{gc} = \begin{bmatrix} \cos(\omega_{gc}t) & -\sin(\omega_{gc}t) & 0 & r(t)\cos(\omega_{gc}t) \\ \sin(\omega_{gc}t) & \cos(\omega_{gc}t) & 0 & r(t)\sin(\omega_{gc}t) \\ 0 & 0 & 1 & z(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Defining ω_g and ω_c as the angular velocity of the workpiece gear and cutter, respectively, and $\omega_{gc} = \omega_g - \omega_c$ is the relative angular velocity between the two. ω_g and ω_c are related by the number of teeth on the cutter (N_c) and gear (N_g): $\omega_g/\omega_c = N_c/N_g$. Reciprocal motion $z(t)$ of the cutter in the vertical direction is generated by a slider-crank mechanism, as shown in Fig. 1d, and has the form:

$$z(t) = a\cos(\omega_s t) + \sqrt{l^2 - a^2\sin^2(\omega_s t)} - l \tag{2}$$

Above, the frequency ω_s is determined from the programmed strokes per minute. The stroke length $2a$ is based on the workpiece width and clearances above and below. The center-to-center distance ($r(t)$) is governed by the radial infeed motion. The most common infeed method is Radial with Rotary (RwR), where the radial feed occurs together with rotational feed. The infeed is defined by the start and end radial feed velocities $v_{r,\mathrm{start}}$, $v_{r,\mathrm{end}}$. Approximating infeed acceleration as piecewise constant ('step') form, the total time to infeed the cutter (t_{infeed}), corresponding acceleration (a_r), and center-to-center distance ($r(t)$) can be found from Eq. (3). This equation is for internal gear workpieces, but can easily be modified for external gears as well.

$$\begin{aligned} t_{\mathrm{infeed}} &= \frac{2(r_{\mathrm{end}} - r_{\mathrm{start}})}{v_{r,\mathrm{end}} - v_{r,\mathrm{start}}}, \quad a_r = \frac{v_{r,\mathrm{end}} - v_{r,\mathrm{start}}}{t_{\mathrm{infeed}}} \\ r(t) &= \begin{cases} r_{\mathrm{start}} + v_{r,\mathrm{start}}t + (1/2)a_r t^2, & t < t_{\mathrm{infeed}} \\ r_{\mathrm{end}}, & r \geq t_{\mathrm{infeed}} \end{cases} \end{aligned} \tag{3}$$

For a single pass process, r_{start} is the radial distance at which the cutter and workpiece barely scrape each other (i.e., a function of their addendum circles). $r_{\mathrm{end}} = r_g - r_c$ is the nominal center-to-center distance for steady-state gear generation. Once the nominal center-to-center distance is reached, the workpiece is rotated an additional 360° to complete the machining pass. In validating the kinematic model, the predicted motion profiles were compared with experimental position/velocity history data from the feed drives of a Liebherr LSE 500 gear shaping machine tool using the Siemens 840D CNC, during the production of different gears. The model was also validated using Liebherr's 'Verzahnanalyse' gear profile prediction software (Fig. 2).

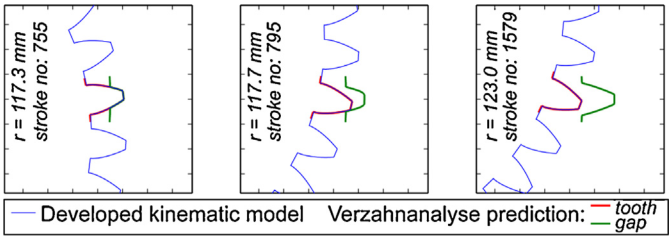


Fig. 2. Verification of kinematic model with 'Verzahnanalyse' software.

3. Cutter-workpiece engagement (CWE)

The shaping process has been simulated using Module Works engine, which uses multi-dexel discrete volume representation. Dexel representation is a method for describing surfaces and volumes, by defining them using parallel line segments (also called 'nails') at discrete intervals. The start and end point of each nail indicate where material begins and ends. Using multiple directions, i.e., x, y, z, to orient nails results in multi-dexel representation, which improves the geometric accuracy [8].

The cutter and workpiece geometries are imported into the engine using Standard Tessellation Language (STL) files, as shown in Fig. 3. Cutter geometry, including the involute profiles of the teeth, is first generated in a solid modeler, then a thin slice at the bottom of the cutter is exported as the STL file. Typically, shaper cutters have side relief angles cut into the gear teeth such that when the cutter is rotating while engaged in the workpiece, the flanks of the teeth do not rub against previously cut surfaces. Using a thin slice of the cutter allows for these relief angles to be omitted from the model, and also decreases the simulation time.

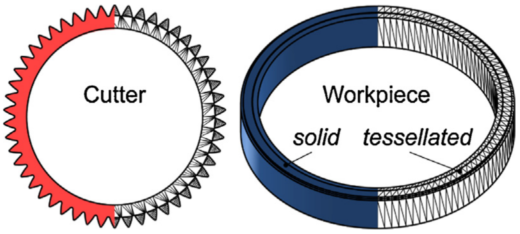
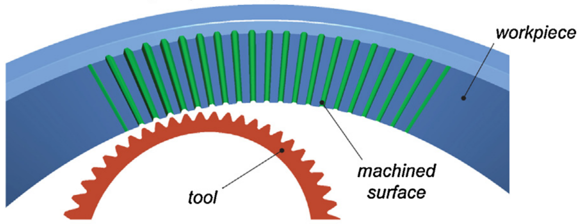


Fig. 3. Solid and tessellated geometry of cutter and workpiece blank.

Visualization of a typical simulation during infeed, steady-state cutting, and completion is shown in Fig. 4, based on parameters summarized in Table 1. The cutting is simulated at discrete time intervals, defined by starting and end tool frames. Each frame is

(a) Radial infeed of shaper (t = 30 [s])



(b) Constant feed (t = 90 [s]) (c) Complete workpiece (t = 276 [s])

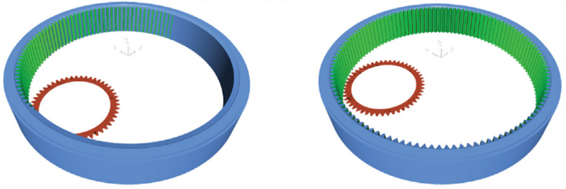


Fig. 4. CWE simulation during infeed, constant feed, and completion.

Table 1
Cutter and workpiece geometry and process parameters.

Module [mm]	Pressure angle [deg]	No. of teeth	Addendum diameter [mm]	Rake angle [deg]	Tip radius [mm]
Cutter geometry					
1.5875	25	50	82.51	5	0.25

Module [mm]	Pressure angle [deg]	No. of teeth	Addendum diameter [mm]	Width [mm]
Workpiece geometry				
1.5875	25	121	188.41	34.85

	(Number of passes: 1)		
Cutting speed [double strokes (DS)/min]	Rotary feed, start [mm/DS]	Radial feed, start [mm/DS]	Radial feed, end [mm/DS]
Process parameters			
350	0.5	0.01	0.01