CIRP Annals - Manufacturing Technology 64 (2015) 137–140

Contents lists available at ScienceDirect

CIRP Annals - Manufacturing Technology

journal homepage: http://ees.elsevier.com/cirp/default.asp

Analytical time-domain turning model with multiple modes

Luke Berglind *, John Ziegert

University of North Carolina at Charlotte, United States

Submitted by Scott Smith (1).

ARTICLE INFO

Keywords:
Turning
Chatter
Time domain model

ABSTRACT

An analytical time domain model is developed to predict the motion of a multi-mode cutting tool during orthogonal turning operations. This model is an extension of a single mode model that finds the solution to the governing delay differential equation (DDE) as a combination of constituent curves (sequential responses) which are independent of the delay term, τ. In the current model, the delay independent constituent curves are found through a recursive state-space solution wherein the individual modal displacements are determined for each sequential response. In this paper, the solution process is described in detail for a two mode system and the resulting analytical time responses are compared with numerical simulations.

1. Introduction

Understanding and characterizing the dynamic characteristics of cutting processes is critical for reducing the negative effects of chatter in machining environments. One of the first successful dynamic models of machining processes was developed by Tobias and Fishwick [1] who established regenerative forces in machining processes as the underlining cause of chatter. This model was later simplified by Tlusty and Polacek [2] who recognized the chip width and the relative tool position normal to the chip width direction as the critical variables for regenerative chatter. In this simplified model, the chip width, b, is assumed to be constant, and the only variation in chip area, and thus the cutting force, is due to variations in the chip thickness, h (assuming orthogonal cutting), as shown in Fig. 1. The resulting linear delay differential equation describing the system dynamics, shown in Eq. (1), allows for stability to be more easily predicted based on the system parameters and the time delay, τ, by comparing amplitudes from one rotation to the next through the system's oriented frequency response function (FRF). Tlusty's approach was then adapted for milling processes by including additional factors to account for varying force direction and number of cutting teeth [2,3].

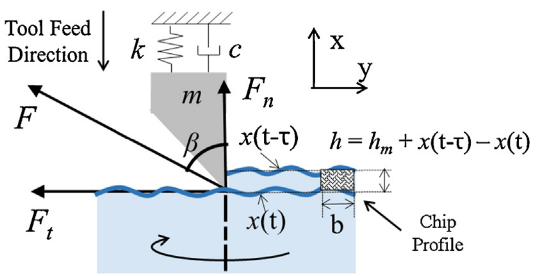

Fig. 1. Orthogonal turning model in which the instantaneous chip area is proportional to the current position of the tool and the tool position in the previous revolution.

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{bK_{s,x}}{m}(h_m + x(t-\tau) - x(t)) \tag{1}$$

Since Tlusty's model much work has focussed on improving the accuracy of stability prediction by including additional nonlinear effects. Some analytic approaches have been developed for more complex conditions, such as temporal finite element analysis [4], semi-discretization methods [5] and multi-frequency solutions [6].

Others have used numerical simulations to determine tool behavior over time, using many simulations to gain a global picture of stability [7]. Numerical simulations are often used to predict tool behavior over time due to their ability to easily account for nonlinearities in the system. However, even for the linear case in Eq. (1) numerical simulations are often used due to the difficulty in determining time history of the tool motion analytically. The most common analytical approach, the method of steps [8], has been employed to solve Eq. (1) for a single degree of freedom system [9]. However, a numerical solution developed for Matlab [10] was eventually used due to the cumbersome nature of the analytical solution.

In this paper, Eq. (1) is evaluated to determine the time history of a tool analytically for the case of multiple modes. The solution developed here is an extension of the solution process developed for a single degree of freedom system in [11], where the total solution is composed of a fixed set of curves. These constituent curves, referred to as sequential responses in [11], all stem from some initial perturbation event and are independent of the time delay term, τ.

* Corresponding author.
E-mail address: lberglin@uncc.edu (L. Berglind).

http://dx.doi.org/10.1016/j.cirp.2015.04.055
0007-8506/© 2015 CIRP.

2. Turning model for multiple modes

The sequential responses for a single degree of freedom system are determined by recursively solving the ordinary differential equation shown in Eq. (2).

$$\ddot{x}_j + \frac{c}{m}\dot{x}_j + \frac{k + bK_{s,x}}{m}x_j = \frac{bK_{s,x}}{m}x_{j-1} \\ x_0(t) = x_{init}(t), \quad x_j(0) = 0, \quad \dot{x}_j(0) = 0 \tag{2}$$

where m is the modal mass, c is the damping coefficient, k is the stiffness, b is the depth of cut (for orthogonal turning), $K_{s,x}$ is the material cutting force coefficient in x-direction, x_j is the current tool point position, x_{j-1} is the solution of the previous sequential response, and x_{init} is the initial input function from which the recursive solutions progress. The responses derived in Eq. (2) are independent of the delay term, τ, however the delay term is accounted for when the sequential responses are combined to form the total solution according to Eq. (3) [11].

For single mode systems, the sequential responses can be iteratively solved directly using Eq. (2) as was done in [11]. For multiple modes, however, intermodal dependencies due to the forcing function require a state space solution. That is, because the total motion of the tool contains components of each individual mode, the instantaneous chip thickness, and thus the force applied to each mode is dependent on the motions of every mode.

$$x(t) = \sum_{j=1}^{N} \begin{cases} 0, & t < (j-1)\tau \\ x_j(t-(j-1)\tau), & t \geq (j-1)\tau \end{cases} \tag{3}$$

Take for example the two DOF linear, orthogonal turning model shown in Fig. 2. Through modal analysis, the total position of the tool can be determined as the summation of the two modal mass positions, assuming that the modal transformation matrix is normalized to x_1, such that, $x_1 = q_1 + q_2$. Although the free response of the system can be determined by analysing the two decoupled modal systems independently, the inclusion of the forcing term, $F(t)$, which is dependent on the position of both modes according to Eq. (4) causes the modal system to be recoupled when the tool is in the cut. The resulting system of equations for the two mode system shown in Eq. (5).

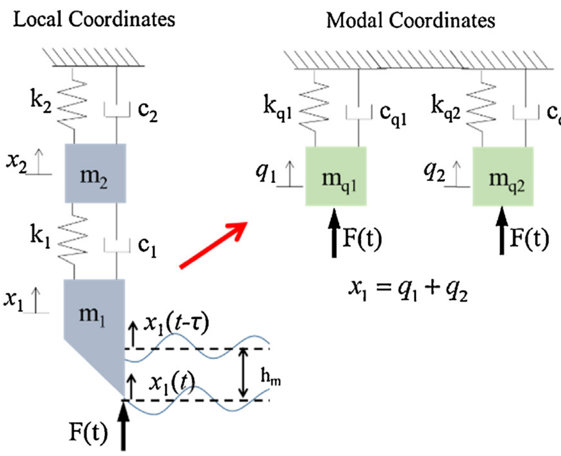

Fig. 2. Two degree of freedom turning model local and modal representation, where the force applied to the two independent modal systems is proportional to the position of the tool point, x_1.

$$F(t) = bK_{s,x}(h_m + x_1(t-\tau) - x_1(t)) \\ = bK_{s,x}(h_m + (q_1(t-\tau) + q_2(t-\tau)) - (q_1(t) + q_2(t))) \tag{4}$$

$$m_{q1}\ddot{q}_1 + c_{q1}\dot{q}_1 + k_{q1}q_1 = bK_{s,x}\left(x_1(t-\tau)\overbrace{-q_1(t) - q_2(t)}^{-x_1(t)}\right) \\ m_{q2}\ddot{q}_2 + c_{q2}\dot{q}_2 + k_{q2}q_2 = bK_{s,x}(x_1(t-\tau) - q_1(t) - q_2(t)) \tag{5}$$

Eq. (5) is evaluated by separating the current time terms, $q(t)$, and the delay terms, $x(t-\tau)$, and replacing the delay terms with a known function, x_{j-1}. The known delay functions are equal to the tool position in the previous solution, where $x_j = q_{1,j} + q_{2,j}$. The resulting recursive expression used to determine the sequential responses for the two degree of freedom system is shown in Eq. (6), where the subscript, j, indicates the jth sequential response. Note that in Eq. (6) the subscript for x_1 is omitted, and the new x subscripts indicate the motion of the tool for each response. In the recursive solutions, the initial conditions are zero for the initial velocities and displacements except for the initial response which are defined by the user, and the history function describing the initial shape of the surface is defined as $x_0 = x_{init}(t)$.

A state space solution is used to solve for the coupled system of equations in Eq. (6). The state variable, r, is defined in terms of the modal positions and velocities $(r_1 = q_1, r_2 = \dot{q}_1, r_3 = q_2, r_4 = \dot{q}_2)$, and the matrix form of the state space equations is shown in Eq. (7)

$$m_{q1}\ddot{q}_{1,j} + c_{q1}\dot{q}_{1,j} + k_{q1}q_{1,j} + bK_{s,x}q_{1,j} + bK_{s,x}q_{2,j} = bK_{s,x}x_{j-1} \\ m_{q2}\ddot{q}_{2,j} + c_{q2}\dot{q}_{2,j} + k_{q2}q_{2,j} + bK_{s,x}q_{1,j} + bK_{s,x}q_{2,j} = bK_{s,x}x_{j-1} \\ q_{1,j}(0) = q_{2,j}(0) = \dot{q}_{1,j}(0) = \dot{q}_{2,j}(0) = 0 \\ q_{1,1}(0) = q_{1,init}, \quad q_{2,1}(0) = q_{2,init}, \quad \dot{q}_{1,1}(0) = \dot{q}_{1,init}, \quad \dot{q}_{2,1}(0) = \dot{q}_{2,init} \\ x_0 = x_{init}(t), \quad x_j = \sum_{n=1}^{N} q_{n,j} = q_{1,j} + q_{2,j} \tag{6}$$

$$\begin{Bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \\ \dot{r}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(k_{q1} + bK_{s,x})}{m_{q1}} & \frac{-c_{q1}}{m_{q1}} & \frac{-bK_{s,x}}{m_{q1}} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-bK_{s,x}}{m_{q2}} & 0 & \frac{-(k_{q2} + bK_{s,x})}{m_{q2}} & \frac{-c_{q2}}{m_{q2}} \end{bmatrix} \times \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{bK_{s,x}}{m_{q1}} \\ 0 \\ \frac{bK_{s,x}}{m_{q2}} \end{Bmatrix} x_{j-1} \\ \{\dot{R}\} = [A]\{R\} + \{F\}x_{j-1} \tag{7}$$

The state space system in Eq. (7) is decoupled by substituting $A = PDP^{-1}$, where P is the eigenvector matrix of A, and D is a diagonal matrix of the eigenvalues of A. After pre-multiplying both sides by P^{-1}, and substituting the variables, $W = P^{-1}R$, and $G = P^{-1}Fx_{j-1}$, the system of equations becomes.

$$\{\dot{W}\} = [D]\{W\} + \{G\} \tag{8}$$

The decoupled system in Eq. (8) can now be used to solve for W, which is then converted back to modal positions to determine the motion of the tool for each sequential response.

Before discussing the solutions to Eq. (8), consider the general process used to determine the sequential responses for the two mode system. In Fig. 3 the recursive solution described in Eq. (6) is

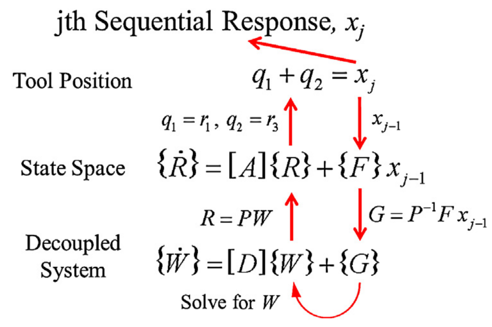

Fig. 3. Diagram showing the recursive solution process used to find the sequential responses for multiple modes, where a new function, x_j, is found during each iteration which is dependent on the previous function, x_{j-1}.