Chaos Theory in Production Scheduling

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Abstract
In this paper the concept of chaos in manufacturing systems is briefly introduced and tools used in the characterization of a chaotic system are discussed. The scheduling of a simple manufacturing system, with the help of commonly used assignment rules, has been simulated first. The results have been studied with the help of phase portraits. Some conclusions have been drawn and a new method for scheduling is proposed. The method is tested against conventional rules and the results are evaluated and discussed.

Keywords:
Scheduling, Control, Decision-making

1 INTRODUCTION

Application of the chaos theory to production systems is considered as a relatively new area [1]. In process type (continuous) production systems, the models considered, usually involve a system of liquid tanks with a switched flow server. In this model, fluid continuously flows out of each tank and the server can fill one tank at a time with a constant rate to compensate for the loss of fluid. Statistical analysis was used over a model of differential equations in order to show the chaotic behavior of such a system [2]. In another approach [3], it was argued that the chaos can be controlled to minimize some cost function of a similar three-funnel model with chaotic behavior and continuous flow. Chaotic behavior of a discrete-event model, involving a paint spraying installation, has been demonstrated in [4]. A model has been investigated, consisting of an order release unit, a buffer, two parallel machines, a switch, and an exit [1]. In another paper [5], where a simple reentrant model is explored with one machine and two basic types of items to be processed, complex but not chaotic behavior was observed in the case of small processing capacity, where the queue grows continuously.

A different approach involves the analysis of the structural stability of a dynamic system, where parameters, such as scheduling rules and WIP-levels, are perturbed to investigate qualitative changes in the behavior of the system [1]. According to [6-8], in some cases, when production becomes very heavily loaded, performance may become unpredictable in unexpected ways: a number of discrete-event simulation models showed that optimum schedules for heavily loaded semiconductor production units, changed dramatically with only slight changes in the input [6, 7].

In a strict theoretical sense, chaotic behavior in simple discrete production models has not been solidly proven [1]. One of the commonly acclaimed characteristics of chaotic behavior, i.e. sensitivity to initial conditions, seems to be present in real manufacturing systems. Empirical evidence on such behavior has been reported [6-8].

Most of the research related to chaos and manufacturing, investigates if and how a manufacturing system exhibits chaotic character. However, there are a number of tools and methods used to detect and measure chaos in a system that can be of interest in studying the behavior of a manufacturing system [9]. A phase space represents all variables of a system. The dimension of the phase space depends on the number of the system's variables. The graphical representation of a phase space, a *phase portrait*, enables the view of a system's behavior in geometric form, without directly representing the variable of time.

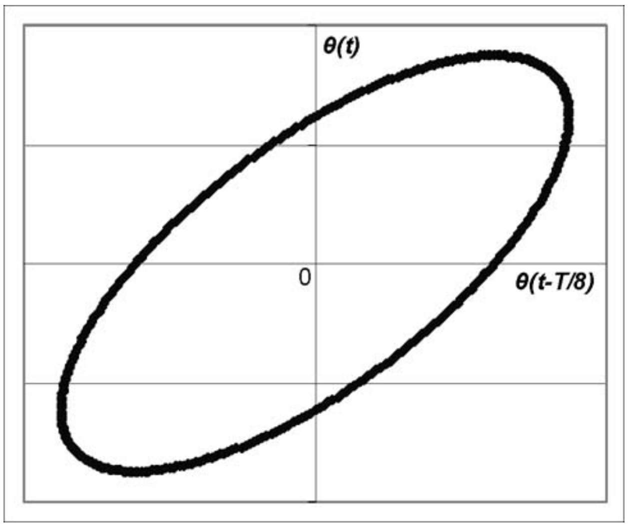

Figure 1: Phase portrait of a pendulum: angular displacement at time t (abscissa) and time t-T/8 (ordinate).

A special subset of this form of graphical representation – often used in nonlinear time series analysis, [10] – plots each value of a data series on the Y-axis (abscissa) against the previous value on the X-axis (ordinate). According to

[11] and [12], such plots capture the topology of the system, for which there is no direct access. In case of a perfect pendulum (Figure 1) one can derive the angular displacement θ at time t versus the angular displacement at a previous point of time t-T/8, where T, the pendulum's period.

The work reported in this paper focuses on the use of phase portraits in studying dispatching rules in a manufacturing system and on devising a different dispatching policy, based on the analysis of phase portraits.

2 DISPATCHING RULES AND PHASE PORTRAITS

In order to examine the behavior of different dispatching policies with the help of phase portraits, a set of experiments has been executed, using three dispatching rules that were selected and applied on a single machine model under heavy processing load. The three rules that were selected were:

- Shortest Processing Time (SPT).
- First-Come-First-Serve (FCFS).
- Earliest Of Due Dates (EODD).

The experiments were conducted by dispatching a string of 5,000 jobs in a single resource according to each rule. The 5,000 jobs were characterized by their arrival time, processing time and due date. The arrival times were randomly generated, following an exponential distribution with mean inter-arrival time of 62.5 time units. The processing times were generated following a normal distribution with a mean value of 100 time units and a standard deviation of 30 time units. Finally, the due dates were calculated following a uniform distribution of random values within the range [0, *PTA*], where *PTA* is the Processing Time Average for all the jobs in the string.

The performance of the rules was evaluated using four performance indicators, namely:

- Mean Tardiness, which, for each job, is given by: T_n=max[0,ET_n–DD_n], where T_n, ET_n and DD_n represent the tardiness, the completion (end) time and the due date of job n, respectively.
- Mean Lateness, which, for each job, is given by: L_n=ET_n–DD_n.
- Mean Flowtime, which, for each job, is given by: F_n=ET_n–AT_n, where AT_n is the arrival time of job n.
- Fraction Tardy, which is the fraction of delayed jobs, expressed as percentage.

For each set of arrival times, processing times and due dates, the experiment was conducted, for statistical reasons, 20 times (i.e. 20 strings of 5,000 jobs) and the mean values of the performance indicators were calculated (Table 1).

	SPT	FCFS	EODD
Mean Tardiness	75,529	94,338	94,338
Mean Lateness	75,529	94,338	94,338
Mean Flowtime	75,579	94,388	94,388
Fraction Tardy	99.0%	99.9%	99.9%

Table 1: Results of scheduling experiments for each rule.

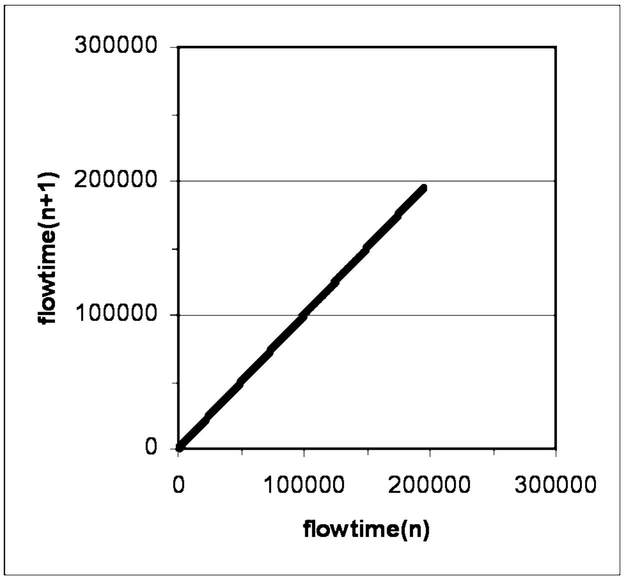

Figure 2: FCFS phase portrait.

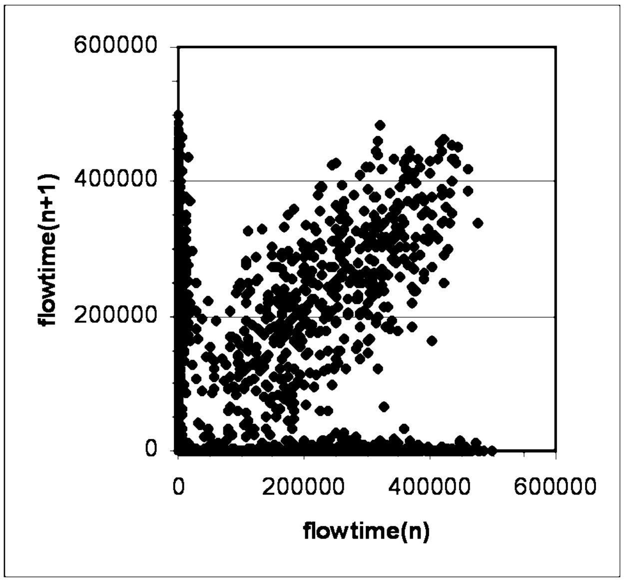

Figure 3: SPT phase portrait.

The results show that SPT in general has a better performance. Following these results a phase portrait was developed for each rule and for the 5,000 jobs, with job flowtime as a variable, where the abscissa is the value of the flowtime of the job n, and the ordinate the flowtime of the job n+1. The lines connecting successive points have been removed from these diagrams in order to facilitate the observation of the spreading of the points across the plane.

One can easily see that in the phase portrait of FCFS (EODD has an almost identical phase portrait) all points are very closely located to the diagonal, meaning that the flowtimes of all the jobs follow a steadily increasing pattern. On the other hand, SPT tends to demonstrate not only a diagonal band of points but also many more points near the two axes, 'producing' some extremely large flowtimes, while in general, the flowtimes are kept rather low. FCFS (and EODD) keep the flowtimes of successive jobs very close to each other, resulting in much lower maximum values, but also 'producing' a rather poor performance.