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Anisotropic plasticity model coupled with strain dependent plastic strain and stress ratios

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ABSTRACT

It is necessary to describe properly anisotropic material behavior for realistic numerical analyses of sheet metal forming processes. The implementation of many yield criteria in finite element analysis is very complicated. Various material tests are also required to determine yield function coefficients. Stress ratios and anisotropy coefficients are not constant during forming processes due to deformation induced anisotropy. This paper introduces a yield function using strain dependent plastic strain ratios and stress ratios. The main advantage is to fully utilize the data of uniaxial tensile tests. The described material behavior shows a significantly improved agreement with experimental data.

1. Introduction

Key aspects of numerical simulation in sheet metal forming processes are the prediction of material failure and wrinkling as well as the improvement of tool die faces for better dimensional fit of the produced sheet metal parts [1]. Especially, it is essential to properly calculate the amount of springback in combination with an effective compensation strategy, see e.g. Yanagimoto et al. [2] or Muthler et al. [3]. Springback depends directly on the stress state at the end of forming process before opening forming tools. Therefore, the effective simulation of such processes requires a proper description of anisotropic plastic material behavior. For this reason many researchers have introduced various yield criteria that describe the plastic anisotropy of sheet metals. Hill [4] proposed a quadratic yield function that fits well the yield surface of cubic metals, especially steel. Hosford [5] proposed a nonquadratic yield function based on crystal plasticity. Barlat et al. [6] developed a six-component yield function extending the Hosford's criterion. Karafillis and Boyce [7] generalized the idea of a linear transformation of the actual anisotropic stress tensor. Banabic and Siegert [8] developed a yield criterion to consider the biaxial anisotropy coefficient. Barlat et al. [9] introduced two linear stress transformations. Cazacu et al. [10] proposed an orthotropic yield criterion to describe asymmetrical yielding effects in uniaxial tensile and compression stress. However, the mathematical expressions of these yield criteria are often so complicated that it is difficult to identify the required material parameters or to implement them into Finite Element code. Moreover, many material tests are needed to determine the yield function coefficients. These phenomenological yield functions typically focus on the description of a constant anisotropic yielding behavior. Hence, the yield function coefficients are constant during the forming analysis, if the yield function is fitted to the yield locus. However, uniaxial stress ratios and plastic strain ratios (denoted as Lankford coefficients or *r* values) are not constant due to deformation induced anisotropy during forming processes. Nowadays improvements in optical measuring systems allow a proper experimental identification of the developing anisotropy even at small strains. According to the direction dependent tensile tests, it is confirmed that the stress ratios and the *r* values are not uniform in size. Furthermore, it is essential to compensate the amount of elastic strain, because this cannot be neglected at the beginning of plastic deformation.

In this paper, the material behavior of a dual phase steel DP800 with thickness of 0.96 mm is described using a yield function considering strain dependent plastic strain ratios and directional stress ratios. For a simple implementation of this initial approach the proposed yield function is based on the Hill's quadratic yield criterion. To reduce experimental complexity, the present article's scope is to fully utilize the results of uniaxial tensile tests on the assumption that the biaxial stress is equivalent to the uniaxial stress in 0° to the rolling direction. Infinite Lankford coefficients for small plastic strains are resolved assuming that a consistent formulation fulfills the associated flow rule at the beginning of plastic strain. A limited initial *r* value on each rolling direction can be computed with the stress ratios and the yield criterion. The plastic strain dependent *r* values are fitted to experimental results with increasing plastic strain. Therefore, it is possible to use all information obtained from the tensile tests in consistency with the selected plasticity model. The proposed approach leads to an improved numerical accuracy, especially for springback analysis and it is verified at 90° bending process.

2. Determination of input parameters

2.1. Flow stress curve

The uniaxial tensile tests are carried out at the universal test machine Typ 1484/DUPS-M ZWICK GmbH. The dog-bone shaped

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tensile specimens are milled according to DIN 50125 H 20 × 80. Tests are conducted under the test conditions after DIN 10002-1 with a quasi-static strain rate of 0.0025/s. During tests the tensile force and strains in the pulling and width direction are measured continuously and repeated under 0°, 45° and 90° to the rolling direction. The directional flow stress curves are extrapolated by Swift isotropic hardening rule [11]:

$$\bar{\sigma}(\bar{\varepsilon}^{pl}) = K(\varepsilon_0 + \bar{\varepsilon}^{pl})^n \tag{1}$$

where K is the strength coefficient, n is the work hardening exponent, ε_0 is the offset strain as well as $\bar{\sigma}$ and $\bar{\varepsilon}^{pl}$ the equivalent stress and true plastic strain, respectively. The parameters of Swift model for each direction are summarized in Table 1.

Table 1
Direction dependent parameters of Swift model.

Angle (°)	K (MPa)	ε_0	n
0	1402.911	0.00367	0.17611
45	1379.203	0.00494	0.18451
90	1399.191	0.18451	0.16811

2.2. *Hill's quadratic yield criterion for plane stress condition*

Hill's quadratic yield function is widely used in practice, because it is easy to handle in analytical and numerical calculations. The yield function for plane stress condition is written by:

$$\bar{\sigma} = \sqrt{G\sigma_{11}^2 + F\sigma_{22}^2 + H(\sigma_{11} - \sigma_{22})^2 + 2N\sigma_{12}^2} \tag{2}$$

where F, G, H and N are Hill's coefficients.

The Hill's coefficients can be determined by uniaxial stress ratios or plastic strain ratios. In both cases the Hill's parameters are conventionally constant during the forming analysis, because they are determined by initial yield stresses or constant Lankford coefficients. However, this assumption of constant anisotropic plastic behavior is not suitable for describing the deformation induced anisotropic plasticity.

In this study, the effective strain dependent Hill's coefficients are introduced based on the uniaxial stress ratios and plastic strain ratios. This approach allows calculating the actual Hill's coefficients on the effective strain. The definitions of two types of the Hill's coefficients are summarized in Table 2. In case of stress ratios based coefficients, the yield stress in normal direction σ_{33} cannot be identified directly by the tensile tests. Thus, it is necessary to perform more complex experiments such as multilayer compression test. To reduce experimental complexity, it is assumed that the stress in normal direction σ_{33} is a mean value of the in-plane yield stresses by:

$$\sigma_{33}(\bar{\varepsilon}^{pl}) = \frac{1}{4}[\sigma_{0^\circ}(\bar{\varepsilon}^{pl}) + 2\sigma_{45^\circ}(\bar{\varepsilon}^{pl}) + \sigma_{90^\circ}(\bar{\varepsilon}^{pl})] \tag{3}$$

where σ_{0°, σ_{45°, σ_{90° are the directional yield stresses under 0°, 45° and 90° to the rolling direction. With this assumption one can determine the parameters F, G, H and N as listed in Table 2 (left). Assuming an associated flow rule the stress ratios can be replaced by the strain ratios as given in Table 2 (right). The determination of the Hill coefficients using strain dependent stress ratios or modeled plastic strain ratios (see Section 2.3) are distinguished by subscripts σ and R, respectively. The plastic strain ratios and yield stress ratios can be resolved by the relation between F, G, H and N using simple algebraic operations:

Table 2
Strain dependent stress ratio and plastic strain ratio based Hill's coefficients under plane stress condition.

Stress ratio based	Plastic strain ratio based
$F_\sigma = \frac{1}{2}\left[\frac{\sigma_{0^\circ}^2(\bar{\varepsilon}^{pl})}{\sigma_{90^\circ}^2(\bar{\varepsilon}^{pl})} + \frac{\sigma_{0^\circ}^2(\bar{\varepsilon}^{pl})}{\sigma_{33}^2(\bar{\varepsilon}^{pl})} - 1\right]$	$F_R = \frac{R_{0^\circ}(\bar{\varepsilon}^{pl})}{R_{90^\circ}(\bar{\varepsilon}^{pl}) \cdot [1 + R_{0^\circ}(\bar{\varepsilon}^{pl})]}$
$G_\sigma = \frac{1}{2}\left[\frac{\sigma_{0^\circ}^2(\bar{\varepsilon}^{pl})}{\sigma_{33}^2(\bar{\varepsilon}^{pl})} + 1 - \frac{\sigma_{0^\circ}^2(\bar{\varepsilon}^{pl})}{\sigma_{90^\circ}^2(\bar{\varepsilon}^{pl})}\right]$	$G_R = \frac{1}{1 + R_{0^\circ}(\bar{\varepsilon}^{pl})}$
$H_\sigma = 1 - G_\sigma$	$H_R = 1 - G_R$
$N_\sigma = 2\frac{\sigma_{0^\circ}^2(\bar{\varepsilon}^{pl})}{\sigma_{45^\circ}^2(\bar{\varepsilon}^{pl})} - \frac{1}{2}(F_\sigma + G_\sigma)$	$N_R = [2R_{45^\circ}(\bar{\varepsilon}^{pl}) + 1] \cdot \frac{[R_{0^\circ}(\bar{\varepsilon}^{pl}) + R_{90^\circ}(\bar{\varepsilon}^{pl})]}{2R_{90^\circ}(\bar{\varepsilon}^{pl}) \cdot [1 + R_{0^\circ}(\bar{\varepsilon}^{pl})]}$

$$\begin{aligned} R_{0^\circ} &= \frac{H}{G}; \quad R_{45^\circ} = \frac{N}{(F+G) - 0.5}; \quad R_{90^\circ} \\ &= \frac{H}{F}; \quad \frac{\sigma_{0^\circ}}{\sigma_{33}}F + G; \quad \frac{\sigma_{0^\circ}}{\sigma_{90^\circ}} = F + H; \quad \frac{\sigma_{0^\circ}}{\sigma_{45^\circ}} \\ &= \frac{1}{2}N + \frac{1}{4}(F+G) \end{aligned} \tag{4}$$

where R_{0°, R_{45° and R_{90° are the Lankford coefficients in 0°, 45° and 90° to the rolling direction.

2.3. *Modeling of variable* R *values*

Lankford coefficients (r values) are defined as the ratio of the plastic strains in width and thickness direction. At the beginning of the uniaxial tensile test r values are infinite, because the plastic strain in normal direction amounts to almost zero. In violation to DIN 10113 the Lankford coefficients in industrial applications are typically determined by ratios of total strains. At total elongations of more than 5% the amount of elastic strain can be neglected. This forming area is one of the general cases in sheet metal forming simulation using constant r values. Based on this, three constant r values of DP800 are determined between the effective strain of 0.05 and 0.10. The constant r values in 0°, 45° and 90° amount to 0.834, 1.131 and 0.896, respectively.

On the other hand, the material behavior is very nonlinear at the boundary between elastic and plastic strain region in tensile test. For this reason the amount of elastic strain cannot be neglected at low plastic strains. Therefore, we propose a different approach. According to the associated flow rule, the Hill's coefficients determined from the initial Lankford coefficients should be consistent with those of the yield stress ratios. Based on this relation and the above defined stress in normal direction, the initial r values can be calculated by the ratio of yield stresses in Eq. (4). With these initial values a simple fitting function is proposed for the modeled Lankford coefficients (denoted by R instead of r), which must fit into the experimental results at a sufficient range of plastic strain:

$$R_\theta(\bar{\varepsilon}^{pl}) = \frac{R_\theta(0) + A_\theta \cdot \bar{\varepsilon}^{pl}}{1 + B_\theta \cdot \bar{\varepsilon}^{pl}} \tag{5}$$

where θ stands for the angle to the rolling direction and $R_\theta(0)$ is the initial plastic strain ratio, A_θ and B_θ are constants that are determined by curve fitting at the plastic strain range from 0.05 to the fracture. The determined parameters are listed in Table 3. In Fig. 1 the calculated R values show a good accordance to the measured data. Subsequently it is possible to predict the variable R values in high strain ranges alike the extrapolation of the flow stress curve.

Table 3
Model parameters of strain dependent R values in three directions.

Angle (°)	$R_\theta(0)$	A_θ	B_θ
0	0.869	55.995	67.691
45	1.064	79.209	69.176
90	1.028	50.308	58.211