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Position-dependent dynamics and stability of serial-parallel kinematic machines

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ABSTRACT

Parallel kinematic machines exhibit strong position-dependent dynamic behavior, which changes the stability and the productive cutting conditions within the machine work volume. In this paper, position-dependency of a hybrid serial-parallel scissor kinematic machine is modeled by synthesizing its substructural reduced order finite element models. The model allows the efficient investigation of position-dependency as an alternative to using time consuming full order finite element models. The predicted position dependency of the machine with the reduced order model is experimentally validated. Stability maps are simulated as a function of machine positions to identify the productivity levels within the machine work volume.

1. Introduction

High-performance machine tools must have consistently high dynamic stiffness over the entire work volume. This is difficult to achieve with traditional serial machine tool construction, since higher stiffness necessarily involves bigger structures, which require bigger, more powerful and costly feed drive motors. Alternatively, nimbler parallel kinematic machine tools with higher dynamic stiffness capabilities have been developed; however, these suffer strong position-dependent dynamics, leading to sluggish performance over large work volumes [1].

To deliver improved dynamic performance over the whole working range of the machine, a prototype hierarchical hybrid serial-parallel scissor kinematic machine has been developed at the Fraunhofer IWU [2], see Fig. 1. Motion is split between the serial parts that envelope large work volumes and the scissor kinematic arrangement that allows delivery of higher dynamic stiffness at the tool center point (TCP) with higher acceleration capabilities [3].

Though the scissor arrangement with struts of fixed lengths is superior to other parallel kinematic arrangements with struts of varying lengths, the scissor kinematics still exhibits strong position-dependent dynamic behavior [1]. The position-varying dynamics lead to position-varying machining stability of the system, which limits the achievable productivity and performance in the whole working range of the machine [4]. The evaluation of the changing stability helps to plan stable machining trajectories by splitting the motion between the serial and parallel parts. In order to exploit the potentially superior dynamic behavior of such a hybrid arrangement it is necessary to consider the position-dependency of the system at the design and development stage.

Position-dependency may be evaluated from measurements on a physical prototype, if available. Alternatively, investigations may be based on a 'virtual prototyping' approach using finite element (FE) models of the machine [5]. However, response analyses for full machine finite element models which are typically on the order of 1,000,000 degrees of freedom (DOFs) or more, is computationally expensive, and can take up significant portion of the total machine tool design and development effort [6]. Moreover, position-dependent analyses for such hybrid serial-parallel machine tools whose flexible bodies undergo simultaneous translation and rotation is not possible with standard multibody simulation codes; and, requires cumbersome adaptive/re-meshing strategies within the FE environment for every new position of the tool in the work volume.

To facilitate rapid assessment of position-dependency for such hybrid machines, this paper offers a computationally efficient position-dependent multibody dynamic model of the machine. A reduced model substructural synthesis approach proposed earlier by the authors in [7] is extended here to model the entire machine tool system; including a novel method to model the parallel struts

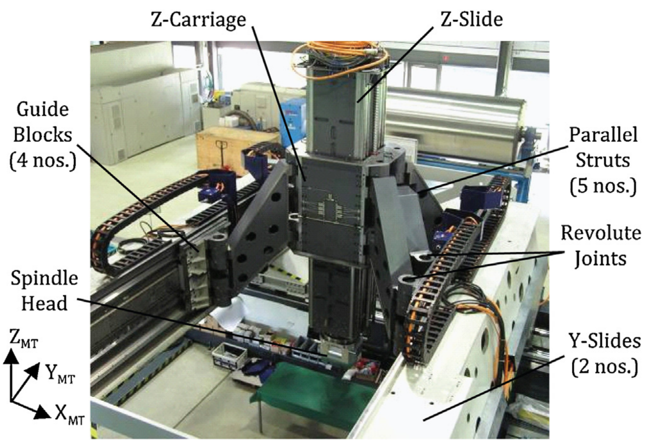

Fig. 1. Hybrid serial-parallel scissor kinematic machine.

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undergoing rotation. The reduced multibody machine model is used to simulate position-dependent stability maps to assess stable material removal rates over the entire work volume.

2. Position-dependent multibody dynamic machine model

The machine is treated as an assembly of the major flexible substructures as shown in Fig. 1. Motion in the *XY* plane is achieved either by four linear motors driving the *Y*-slides, or, through actuating the scissor-kinematic arrangement as shown in Fig. 2. Struts at either side of the platform are driven by action of four independent linear motors, whose planar motion results in rotation of the struts about their pivot points. Tool motion in the *XZ* plane is achieved by two ball-screw feed drives.

During positioning, the substructures undergo simultaneous translation and rotation, which leads to position dependent structural dynamics. First, each major substructure is modeled independently in the FE environment; and is exported, after convergence checks to the MATLAB environment for reduction and synthesis with the parallel struts. The parallel struts in turn are modeled with Timoshenko beam elements which are oriented to the appropriate configuration using transformation matrices prior to assembly with the main structural components.

The details of general mathematical modeling of sub-structuring based on improved reduced order models are given in [7], and only the principles that are specific to the hybrid machine are given here.

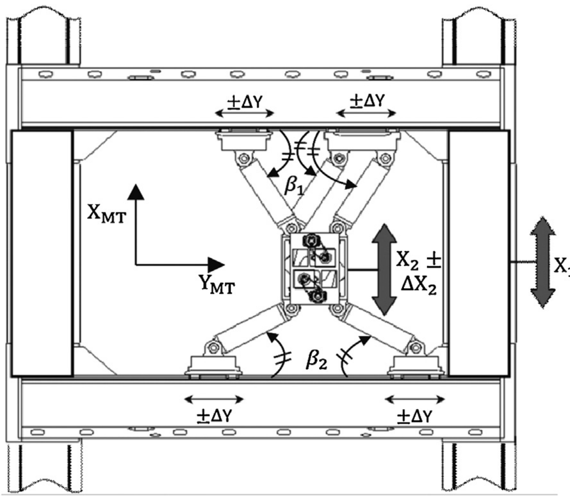

Fig. 2. Kinematic scheme for the scissor kinematics.

2.1. Substructure model reduction

For any structural substructure under consideration, the undamped equations of motion are represented as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \tag{1}$$

where $\{\boldsymbol{M},\boldsymbol{K}\}_{F\times F}$ are the mass and stiffness matrices for the total F number of DOFs and $\boldsymbol{f}_{F\times 1}$ is the force vector. Reduced substructures are synthesized at their interface DOFs; hence, the displacement vector $\boldsymbol{u}$ is partitioned into the DOFs to be retained, i.e. the exterior/interface (E) DOFs, $\boldsymbol{u}_E$, that are in physical contact with the other substructure(s); and, the DOFs which are to be eliminated, i.e. the interior (I) DOFs, $\boldsymbol{u}_I$.

The displacement vector $\boldsymbol{u}$ is expressed in terms of a reduced set, $\boldsymbol{u}_{R_{R\times 1}}$, by a transformation matrix $\boldsymbol{T}_{F\times R}$ (R = F) as:

$$\boldsymbol{u}_{F\times 1} = \boldsymbol{T}_{F\times R}\boldsymbol{u}_{R_{R\times 1}} \tag{2}$$

where

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I}_{EE} & 0_{EP} \\ \boldsymbol{T}_{ICMS_{IE}} & \boldsymbol{\Phi}_{IP} \end{bmatrix} \tag{3}$$

$\boldsymbol{I}_{EE}$ is a unit matrix, and, $\boldsymbol{T}_{ICMS}$ is an improved form of the standard component mode synthesis (CMS) transformation matrix. Essential dynamics from the full model are retained by selecting a subset of significant P modes, i.e. $\Phi_{IP} \subseteq \Phi_{II}$ in Eq. (3); wherein the eigenvector Φ_{II} is obtained by solving the eigenvalue problem corresponding to the interior DOFs [7].

Using the transformation matrix (Eq. (3)), the reduced substructural matrices $\{\boldsymbol{M}_R, \boldsymbol{K}_R\}_{R\times R}$ are represented as:

$$\boldsymbol{M}_R = \boldsymbol{T}^T\boldsymbol{M}\boldsymbol{T};\ \boldsymbol{K}_R = \boldsymbol{T}^T\boldsymbol{K}\boldsymbol{T};\ \boldsymbol{f}_R = \boldsymbol{T}^T\boldsymbol{f}. \tag{4}$$

These substructures are combined with the parallel struts which are modeled separately as discussed below.

2.2. Modeling parallel struts

Each parallel strut is modeled with Timoshenko beam elements that have six DOFs on each of its two nodes; three translational (u_x, u_y, u_z) and three rotations ($\theta_x, \theta_y, \theta_z$) as shown in Fig. 3. A two stage transformation is carried out at the elemental level: i.e. first a rotation about the X axis ($\boldsymbol{R}_x(\alpha)$) to bring the strut to the correct inclination as in Fig. 1; and, a subsequent rotation about the Z axis ($\boldsymbol{R}_z(\beta)$) to make the correct orientation as in Fig. 2. Transformations are based on kinematic relationships.

For a given tool position, the platform subtends different angles at either side ($\beta_1 \neq \beta_2$). The inclination (α) for each of the struts is fixed, i.e. it does not change as a function of tool position. The formulation discussed here is for the general case for any of the five parallel struts; hence, the subscript is dropped.

The transformed structural matrices $\{\boldsymbol{M}_{St_{OR}}, \boldsymbol{K}_{St_{OR}}\}$ for each of the parallel struts at the elemental level are expressed as:

$$\boldsymbol{M}_{St_{OR}} = \boldsymbol{R}_{xz}^T\boldsymbol{M}_{St}\boldsymbol{R}_{xz}; \quad \boldsymbol{K}_{St_{OR}} = \boldsymbol{R}_{xz}^T\boldsymbol{K}_{St}\boldsymbol{R}_{xz} \tag{5}$$

where $\{\boldsymbol{M}_{St}, \boldsymbol{K}_{St}\}$ are the elemental matrices in *YZ* plane; and $\boldsymbol{R}_{xz}$ is the two stage transformation matrix expressed as:

$$\boldsymbol{R}_{xz} = \boldsymbol{R}_x(\alpha)\cdot\boldsymbol{R}_z(\beta) \tag{6}$$

where

$$\boldsymbol{R}_X = \begin{bmatrix} \boldsymbol{R}_{X_j} & 0_{6\times 6} \\ 0_{6\times 6} & \boldsymbol{R}_{X_{j+1}} \end{bmatrix}; \quad \boldsymbol{R}_Z = \begin{bmatrix} \boldsymbol{R}_{Z_j} & 0_{6\times 6} \\ 0_{6\times 6} & \boldsymbol{R}_{Z_{j+1}} \end{bmatrix}. \tag{7}$$

wherein the nodal operator at each node j is expressed as:

$$\boldsymbol{R}_{X_j} = \boldsymbol{R}_{X_{j+1}} = \begin{bmatrix} \boldsymbol{T}_X & 0_{3\times 3} \\ 0_{3\times 3} & \boldsymbol{T}_X \end{bmatrix}; \quad \boldsymbol{R}_{Z_j} = \boldsymbol{R}_{Z_{j+1}} = \begin{bmatrix} \boldsymbol{T}_Z & 0_{3\times 3} \\ 0_{3\times 3} & \boldsymbol{T}_Z \end{bmatrix};$$
$$\text{where } \boldsymbol{T}_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}; \quad \boldsymbol{T}_Z = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

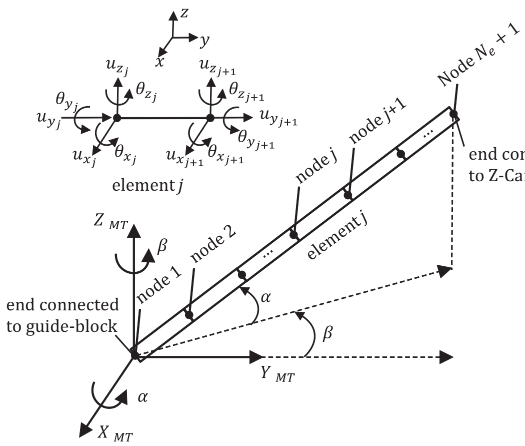

Fig. 3. Oriented parallel strut modeled with Timoshenko beams.