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Time Minimum Trajectory Planning of a 2-DOF Translational Parallel Robot for Pick-and-place Operations

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Abstract
This paper deals with the time-minimum trajectory planning of a 2-DOF translational parallel robot named the Diamond for rapid pick-and-place operations. Kinematics and dynamics of the robot are formulated using a parametric function, allowing the representation of the input torque and velocity constraints to be converted to those in terms of the path length. A modified algorithm for achieving the minimized traversal time is proposed by taking into account the path jerk limit. Lithium-ion battery sorting using the Diamond robot is taken as an example to demonstrate the applicability of this approach.

Keywords:
Parallel Robot, Motion Planning, Pick-and-place

1 INTRODUCTION

It has been acknowledged that the parallel robots driven by proximal arms exhibit great potential for high-speed pick-and-place operations in many sectors such as food, electronics and many other light industries [1]. This statement can be justified by various successful applications of the Delta robot and the likes [2-4].

In order to achieve high productivity, it is desirable for a pick-and-place robot to follow a specified geometric path that gives the minimum time motion. This demand leads to the well-known time-optimal trajectory planning problem which can be resolved either in task coordinates or joint coordinates. Bobrow [5] and Shin [6] independently initiated a method by which the maximum admissible path velocity is determined and shown in phase plane, allowing the path traversal time to be minimized subject to the input torque limits. This method was then amended by others [7,8] via adding various constraints in either joint and/or task coordinates. As an alternative to the task-space-based methods, the problem can be tackled by parameterising the joint motion directly using cubic or higher order polynomials [9].

This paper deals with the time-minimum trajectory planning of a 2-DOF translational parallel robot named the Diamond (see Figure 1). The input constraints due to the limits in joint torque and velocity are considered. Geometric path for pick-and-place operations is generated using the piecewise fifth order polynomial. On the basis of the previous work [5-8], a modified trajectory planning algorithm is proposed by taking into account the path jerk limitation. Lithium-ion battery sorting using the Diamond robot is taken as an example to demonstrate the applicability of this approach.

2 ROBOT DYNAMICS AND PARAMETERISATION

Inverse dynamics of the Diamond robot is governed in a hybrid form [4] by:

$$\boldsymbol{\tau} = \boldsymbol{D}(\boldsymbol{r})\boldsymbol{a} + \boldsymbol{H}(\boldsymbol{r},\boldsymbol{v})\boldsymbol{v} + \boldsymbol{g}(\boldsymbol{r}) \tag{1}$$

where $\boldsymbol{\tau}$ is the joint toque; $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ are the position, velocity and acceleration of the reference point of the end-effector; $\boldsymbol{D}$ is the hybrid inertial matrix, $\boldsymbol{H}$ is the hybrid Coriolis matrix and $\boldsymbol{g}$ is the vector of gravity. The kinematics of the robot can be formulated via inverse kinematic analysis as follows.

$$\boldsymbol{q} = \boldsymbol{p}(\boldsymbol{r}),\ \dot{\boldsymbol{q}} = \boldsymbol{J}\boldsymbol{v},\ \ddot{\boldsymbol{q}} = \boldsymbol{J}\boldsymbol{a} + \dot{\boldsymbol{J}}\boldsymbol{v} \tag{2}$$

where $\boldsymbol{J}$ is the Jacobian matrix and $\dot{\boldsymbol{J}}$ is its time derivative. For more details about kinematic and dynamic formulation of the Diamond robot please refer to [4].

In pick-and-place operations, the geometric path can be generated in a parameterised form in terms of path length s [5,6] (also see Section 4) such that

$$\boldsymbol{r} = \boldsymbol{f}(s),\ \boldsymbol{v} = \boldsymbol{f}'\dot{s},\ \boldsymbol{a} = \boldsymbol{f}'\ddot{s} + \boldsymbol{f}''\dot{s}^2 \tag{3}$$

where $\boldsymbol{f}$ is the parametric function mapping $\boldsymbol{r}$ onto the tangential direction of the path. Thus, the inverse dynamics along the path can be converted to the form

$$\boldsymbol{\tau} = \boldsymbol{a}_2(s)\ddot{s} + \boldsymbol{a}_1(s)\dot{s}^2 + \boldsymbol{a}_0(s) \tag{4}$$

where $\boldsymbol{a}_2 = \boldsymbol{D}\boldsymbol{f}'$, $\boldsymbol{a}_1 = \boldsymbol{D}\boldsymbol{f}'' + \boldsymbol{H}\boldsymbol{f}'$, $\boldsymbol{a}_0 = \boldsymbol{g}$

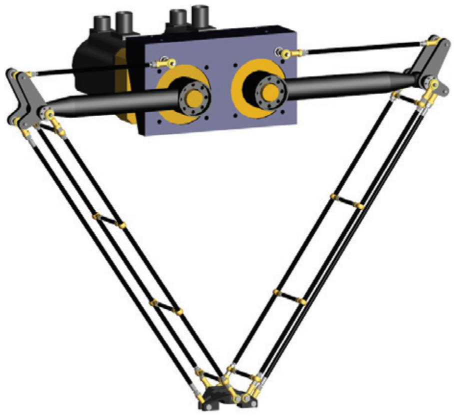

Figure 1: The Diamond robot.

3 CONSTRAINTS

In general, the constraints imposed upon the motion of a robotic manipulator are: (1) the system constraints due to the limits in joint torques and velocities, and (2) the task constraints due to the limits of velocity, acceleration and even jerk of the end-effector. For pick-and-place operations, the system constraints are the dominant factors and they are thereby merely taken into account in this article. Considering two servomotors of the Diamond robot to be identical, the joint torque constraints can be expressed as

$$|\tau_i| \le \tau_{\max}, \quad i=1,2 \tag{5}$$

where $\tau_{\max}$ denotes the maximum admissible joint torque. Hence, the bounds in terms of path acceleration $\ddot{s}$ due to the joint torque limit can be expressed by

$$\ddot{s}^{\tau}_{\min,i}(s,\dot{s}) \le \ddot{s} \le \ddot{s}^{\tau}_{\max,i}(s,\dot{s}), \quad i=1,2 \tag{6}$$

where

$$\ddot{s}^{\tau}_{i,\max} = \begin{cases} a_{2,i}^{-1}\left(\tau_{\max} - a_{1,i}\dot{s}^2 - a_{0,i}\right) & a_{2,i} > 0 \\ a_{2,i}^{-1}\left(-\tau_{\max} - a_{1,i}\dot{s}^2 - a_{0,i}\right) & a_{2,i} < 0 \\ \infty & a_{2,i} = 0 \end{cases}$$
$$\ddot{s}^{\tau}_{i,\min} = \begin{cases} a_{2,i}^{-1}\left(-\tau_{\max} - a_{1,i}\dot{s}^2 - a_{0,i}\right) & a_{2,i} > 0 \\ a_{2,i}^{-1}\left(\tau_{\max} - a_{1,i}\dot{s}^2 - a_{0,i}\right) & a_{2,i} < 0 \\ -\infty & a_{2,i} = 0 \end{cases} \tag{7}$$

It is easy to prove that Eq.(7) will produce a pair of inequalities

$$A\dot{s}^2 + B + C \ge 0, \quad -A\dot{s}^2 - B + C \ge 0 \tag{8}$$

where

$$A = a_{2,1}^{-1}a_{1,1} - a_{2,2}^{-1}a_{1,2}, \ B = a_{2,1}^{-1}a_{0,1} - a_{2,2}^{-1}a_{0,2}, \ C = \tau_{\max}\left(\left|a_{2,1}^{-1}\right| + \left|a_{2,2}^{-1}\right|\right)$$

For a given s, each inequality constitutes an interval and their intersection forms the admissible velocity region within which the path velocity must lie. Considering $\dot{s} > 0$, the maximum admissible path velocity due to the joint torque constraint can therefore be obtained by

$$\dot{s}^{\tau}_{\max} = \min\left[\left(\left|A\right|^{-1}\left|B \pm C\right|\right)^{1/2} \quad \forall A(B \pm C) < 0\right] \tag{9}$$

Similarly, given the maximum admissible joint velocity, $\dot{q}_{\max}$, the path velocity $\dot{s}$ has to be bounded by

$$\dot{s} \le \dot{s}^{v}_{\max} \tag{10}$$

where $\dot{s}^{v}_{\max}$ is the maximum admissible path velocity which can be obtained via inverse kinematics

$$\dot{s}^{v}_{\max} = \dot{q}_{\max} \min_{i=1,2}\left(\frac{1}{\left|\boldsymbol{J}_i\boldsymbol{f}'\right|}\right), \quad i=1,2 \tag{11}$$

where $\boldsymbol{J}_i$ is the ith row of $\boldsymbol{J}$. Thus, the maximum path velocity curve (MPVC) can be determined by

$$\dot{s}_{\max} = \min\left(\dot{s}^{\tau}_{\max}, \dot{s}^{v}_{\max}\right) \tag{12}$$

4 GEOMETRIC PATH GENERATION

Figure 2 shows a typical geometric path for planar pick-and-place operations. Without losing generality, the path contains five segments, i.e. two vertical, one horizontal and two curved segments in between. It should be noted that a vertical segment is needed whenever an object is being picked from or being placed into a hole. Meanwhile, the curved segment should be generated in such a way that at least C^2 continuity is ensured at two extreme points connecting the straight line segments.

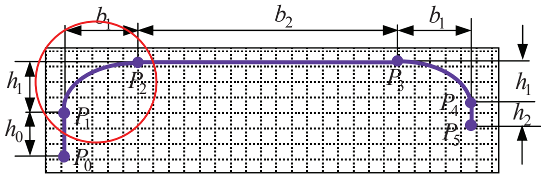

Figure 2: Geometric path for pick-and-place operation in a plane.

Figure 3 shows the curved segment linking the left vertical line with the horizontal one at $\hat{P}_0$ and $\hat{P}_1$. The coordinates of a point on the curve can be expressed by fifth order polynomials in terms of path parameter $\hat{s}$ evaluated in the local coordinate system $\hat{P}_0 - \hat{x}\hat{y}$, i.e.

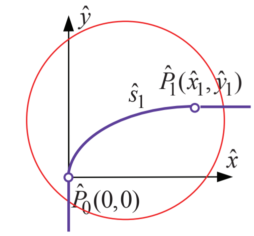

Figure 3: The curved path connecting two orthogonal lines.

$$\hat{x}(\hat{s}) = \sum_{i=0}^{5} a_i\hat{s}^i, \quad \hat{y}(\hat{s}) = \sum_{i=0}^{5} b_i\hat{s}^i, \quad 0 \le \hat{s} \le \hat{s}_1 \tag{13}$$

In order to achieve C^2 continuity at $\hat{P}_0$ and $\hat{P}_1$, the following boundary conditions should be imposed

$$\hat{x}(0)=0, \ \hat{y}(0)=0, \ \hat{x}(\hat{s}_1)=\hat{x}_1, \ \hat{y}(\hat{s}_1)=\hat{y}_1$$
$$\hat{x}'(0)=0, \ \hat{y}'(0)=1, \ \hat{x}'(\hat{s}_1)=1, \ \hat{y}'(\hat{s}_1)=0$$
$$\hat{x}''(0)=0, \ \hat{y}''(0)=0, \ \hat{x}''(\hat{s}_1)=0, \ \hat{y}''(\hat{s}_1)=0$$

This results in a set of unique coefficients as follows:

$$a_0 = a_1 = a_2 = b_0 = b_2 = 0, \quad b_1 = 1, \quad \boldsymbol{Aa} = \boldsymbol{c}, \quad \boldsymbol{Ab} = \boldsymbol{d} \tag{14}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \hat{s}_1^2 & \hat{s}_1 & 1 \\ 5\hat{s}_1^2 & 4\hat{s}_1 & 3 \\ 10\hat{s}_1^2 & 6\hat{s}_1 & 3 \end{bmatrix}, \ \boldsymbol{a} = \begin{pmatrix} a_5 \\ a_4 \\ a_3 \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_5 \\ b_4 \\ b_3 \end{pmatrix}, \ \boldsymbol{c} = \begin{pmatrix} \hat{x}_1/\hat{s}_1^3 \\ 1/\hat{s}_1^2 \\ 0 \end{pmatrix}$$
$$\boldsymbol{d} = \begin{pmatrix} (\hat{y}_1 - \hat{s}_1)/\hat{s}_1^3 \\ -1/\hat{s}_1^2 \\ 0 \end{pmatrix}, \quad \hat{s}_1 = \int_C \sqrt{\left(\frac{d\hat{x}}{d\hat{s}}\right)^2 + \left(\frac{d\hat{y}}{d\hat{s}}\right)^2}\, d\hat{s}$$

It can be proven that $\hat{s}$ is the path length if $\hat{x}_1 = \hat{y}_1$. Similarly, the polynomial coefficients associated with the curved segment linking the horizontal line with the right vertical one can also be obtained by simply replacing

$$\boldsymbol{c} = \begin{pmatrix} (\hat{x}_1 - \hat{s}_1)/s_1^3 \\ 1/\hat{s}_1^2 \\ 0 \end{pmatrix}, \ \boldsymbol{d} = \begin{pmatrix} \hat{y}_1/\hat{s}_1^3 \\ -1/\hat{s}_1^2 \\ 0 \end{pmatrix}, \ a_1 = 1, \ b_1 = 0$$