Applied Mathematical Modelling 40 (2016) 2514–2528

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Acoustic radiation analysis for a control domain based on Green's function

Luyun Chen, Xiaofeng Liang*, Hong Yi

State Key Laboratory of Ocean Engineering, Shanghai Jiaotong University, Shanghai 200240, PR China

ARTICLE INFO

Article history:
Received 11 February 2014
Revised 20 July 2015
Accepted 23 September 2015
Available online 19 October 2015

Keywords:
Green's function
Conformal transformation method
Boundary element method (BEM)
Acoustic directivity
Acoustic radiation power

ABSTRACT

The acoustic radiation problem in the control domain with complex boundaries is investigated. Acoustic Green's function for acoustic point sources in the control domain was obtained by using conformal transformation theory. The expression of the acoustic radiation function for the control domain was derived. The acoustic radiation characteristics of a pulsating sphere in a quarter-infinite domain and numerical simulations were compared to illustrate the effects of the boundary characteristic, location, and radiation frequency on acoustic radiation power and acoustic directivity. This study provides a new method to analyze the acoustic radiation problem with complex boundaries.

1. Introduction

The structural-acoustic radiation problem is a hot issue in marine, aerospace, and automotive engineering design and manufacturing. Three main methods are used for structural-acoustic radiation analysis, namely, theoretical analysis, numerical calculation, and experimental investigation. In the numerical analysis method, the boundary element method (BEM) is one representative method and is an efficient tool for structural-acoustic radiation analysis by reducing the acoustic domain dimensions to one. When an appropriate acoustic Green's function is adopted, the radiation condition at infinity will be automatically satisfied to avoid the artificial boundary.

The Helmholtz equation is a basic equation in acoustic radiation analysis. In most acoustic radiation research, this equation is used to address the acoustic problem in free-space. However, in many practical acoustic problems, the acoustic boundary conditions may be irregular or complicated, and the acoustic radiation may be affected by the boundary characteristics. For example, if the acoustic source is close to the rigid boundary, this boundary will definitely affect the acoustic radiation. The same situation applies for the compliant boundary. Generally, the acoustic wave will be reflected on the rigid boundary surface (zero normal velocity) and the acoustic pressure will be dismissed on the compliant boundary surface (zero acoustic pressure). Moreover, the acoustic radiation in an ocean environment will be distorted by the seabed or water surface. Thus, the influence of boundary characteristics should be analyzed for the acoustic radiation problem in the control domain.

Many studies have been performed on the essential solution of Green's function to address the acoustic radiation problem with complex boundaries. Al-Khaleefi et al. [1] analyzed Green's function for a harmonic acoustic point source within seawater overlying a saturated poroelastic of seabed. Bapat et al. [2] discussed the modified Green's function for acoustic wave propagation in the shallow water domain. By using the fast multi-pole BEM method, the acoustic wave problems of the 3D half-space domain

* Corresponding author. Tel.: +86 21 34207165.
E-mail addresses: cluyun@sjtu.edu.cn (L. Chen), liang_xiaofeng@sjtu.edu.cn (X. Liang).

http://dx.doi.org/10.1016/j.apm.2015.09.019

was analyzed [3]. By using the free-space acoustic Green's function, Li and Huang [4] solved the acoustic absorption problems with the finite element method and far-field computations. The cylindrical structural-acoustic radiation problems were discussed [5], and the influence of rigid boundary or compliant boundary in the half-space domain was compared. Lu and Jeng [6] established a Green's function for an acoustic source within the half-space seawater overlying a porous seabed, which is considered a saturated porous medium. The influence of soft and rigid boundaries on the structural vibration and acoustic radiation problem was investigated [7], and several relative factors including plate thickness, structure damping, and rigid surface on the structural natural frequencies were compared. McGowan and Howe [8] discussed the acoustic radiation of a pulsating sphere in the complex acoustic field domain, including the acoustic radiation directivity effect by the boundary characteristic, radiation frequency, and acoustic source location. Panza [9] discussed the Euler–Maclaurin sum formula, wherein Green's transfer function expression was deduced. Nevels and Jeong [10] provided a brief tutorial discussion on the method of compact Green's function applied to a source of sound in the vocal tract.

Conformal transformation involves the use of the mathematical properties of the analytical complex functions of a complex variable. The conformal transformation method is a popular tool for solving the problem in Green's function problems. Santiago and Wrobel [11] studied the static Green's function by using conformal transformation. Yılmaz et al. [12] applied conformal transformation theory and the finite element method to analyze the electric impedance tomography problem of elliptic geometry. Zou et al. [13] focused on the effect of compliant boundary or rigid boundary to the structural vibration and acoustic radiation problems.

In this article, the acoustic Green's function and acoustic radiation problem in the control domain were analyzed. The modified acoustic Green's function in the control domain was obtained by introducing conformal transformation theory. The acoustic radiation function in the control domain was derived on the basis of the modified acoustic Green's function. In the end, the acoustic radiation power and acoustic directivity problem in the control domain were discussed. On the basis of the acoustic radiation problem of the quarter-infinite domain, the influence of the acoustic radiation power, and the acoustic directivity caused by the boundary characteristics, the acoustic source location and radiation frequency were compared.

The rest of this paper is organized as follows. The acoustic Green's function problems are reviewed in Section 2. The conformal transformation of the modified acoustic Green's function is discussed in Section 3. The acoustic radiation function of quarter-infinite domain is established in Section 4. The acoustic radiation power and acoustic directivity problems in the quarter-infinite domain are analyzed in Section 5. To illustrate the validity of the proposed method, the numerical results and comparisons of the acoustic radiation powers and pressure directivity are presented in Section 6. Finally, the conclusion is drawn in Section 7.

2. Acoustic Green's function

In this section, the acoustic radiation problem was analyzed on the basis of a series of appropriate assumptions. The acoustic medium was assumed inviscid and ideally compressible; thus, the medium cannot support the shear stresses, the state of stress in the fluid is purely hydrostatic, and the acoustic wave equation is a linear equation. Furthermore, only low frequency and middle frequency acoustic radiation problems were discussed in this article.

2.1. Acoustic Helmholtz equation

For the acoustic radiation problem, the acoustic source is usually assumed to be a time harmonic. The principal part of acoustic Green's function is the solution for a conventional acoustic point source in an infinite medium domain. The complementary part corresponding to the general solution of the Helmholtz equation is expressed as follows:

$$\nabla^2 \boldsymbol{p} + \sigma^2 \boldsymbol{p} = 0, \tag{1}$$

where ∇^2 is the Laplace operator, $\boldsymbol{p}(x, y, z, t)$ is the acoustic pressure in the field point, σ is the acoustic wave number $\sigma = \omega/c_0$, ω is the angular frequency of the acoustic wave, and c_0 is the speed of sound with constant.

Suppose that an acoustic source point exists in the acoustic field, the acoustic Green's function for the free-space domain is determined by the following inhomogeneous Helmholtz governing equation:

$$\nabla^2 \boldsymbol{p} + \sigma^2 \boldsymbol{p} = 4\pi \delta(\boldsymbol{r} - \boldsymbol{r}_0), \tag{2}$$

where $\delta(\boldsymbol{r} - \boldsymbol{r}_0)$ denotes the Dirac delta generalized function, $\boldsymbol{r}_0$ is the vector distance between the zero point and acoustic source point, $\boldsymbol{r}$ is the vector distance between the zero point and observer position, and $\boldsymbol{r} - \boldsymbol{r}_0$ is the vector distance between the acoustic observer position and acoustic source-point position.

The solution of Eq. (2) consists of two components: one is the homogeneous solution, which is determined by Eq. (1) and is called the complementary part (regular part); the other is the special solution, which is given by Eq. (2) and is called the principal part (singular part). In Eq. (2), the acoustic Green's function for the principal part has the following form:

$$\boldsymbol{G}(\boldsymbol{r}|\boldsymbol{r}_0) = \frac{\mathrm{e}^{\mathrm{i}\sigma|\boldsymbol{r}-\boldsymbol{r}_0|}}{4\pi|\boldsymbol{r} - \boldsymbol{r}_0|}, \tag{3}$$

where $\mathrm{i} = \sqrt{-1}$.