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An inner annular crack in a superconducting cylinder and its crack front properties under applied magnetic field

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ABSTRACT

An inner annular crack in an infinitely long superconducting cylinder is modeled, and its fracture properties under electromagnetic forces are investigated. First, the flux density distributions in the superconducting cylinder of the present crack model are obtained analytically for both zero-field cooling (ZFC) and field cooling (FC) activation processes, where the magnetically impermeable crack surface condition and the Bean model outside the crack influence region are adopted. Then the stress intensity factors (SIFs) at both inner and outer crack fronts are calculated by the finite element method (FEM). Some important phenomena are observed. Among others, in the process of field ascent, the state corresponding to the applied maximal magnetic field is the most critical. In the process of field descent, for the FC case, the crack always propagates and grows from the inner crack front, and it is easier to propagate when the crack is close to the cylinder's center. For the ZFC case, although the crack generally propagates from the inner crack front as well, whether the crack would propagate or not depends on the crack position, crack size and the applied field. The crack is generally easier to propagate for the FC process than for the ZFC process. The findings from this study are very useful for detailed analysis and design of superconducting materials.

1. Introduction

Crack problems for high-temperature superconductors (HTSs) have been widely investigated [1–7]. In the past decade, many studies [8–12] were carried out to investigate the fracture properties of superconducting slabs and/or cylinders using either analytical method [8] or finite element method (FEM) [9–14]. However, all the cracks mentioned above were straight [8–13] or piecewise straight [14]. Recently, Gao et al. [15] carried out the fracture analysis for a penny-shaped crack problem of a superconducting cylinder, where the crack was not, as before, assumed to be fully penetrated through the thickness of HTSs.

On the other hand, the geometry of internal flat annular cracks in HTS is of practical importance in fracture analysis, since it represents an idealization of either a flattened toroidal crack [16] or a banana-shaped crack [17], which are inherent presence in many intelligent material systems and structures [18–23]. However, the problem of an annular crack in HTS has never been reported in the existing literatures. Thus, in this paper, we further carry out the fracture analysis for superconducting cylinder with a novel crack model, i.e., with an inner annular crack. Different from [15], a new crack influence region around the pre-existing annular crack in HTS is introduced. The flux density distributions in the superconducting cylinder under the present

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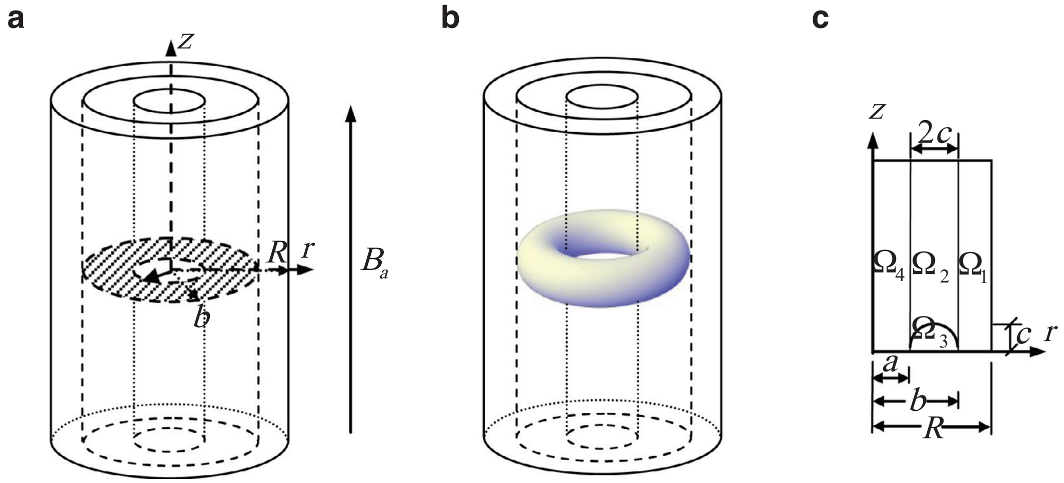

Fig. 1. An inner annular crack in an infinitely long superconducting cylinder subjected to a parallel magnetic field: (a) mathematical model, (b) schematic diagram of crack influence region, (c) four different regions in a quarter of longitudinal symmetry plane.

Nomenclature

r, θ, z	coordinates in the cylindrical coordinate system
R	the radius of the superconducting cylinder
a	inner radius of the crack
b	outer radius of the crack
μ_0	magnetic permittivity
J_c	critical current density
r_0	position where the remagnetization just arrives at
B_a	applied magnetic field
B	magnetic field in the superconducting cylinder
B_p	full penetration field of a superconducting cylinder without cracks
B_1^*, B_2^*, B_3^*	external characteristics fields corresponding to $r_0 = b$, $r_0 = a$ and $r_0 = 0$, respectively
B_m	maximum applied magnetic field in the case of ZFC process
B_{fc}	maximum applied magnetic field in the case of FC process
K_{Ia}, K_{Ib}	Mode-I SIFs at the crack fronts $r = a$ and $r = b$, respectively
$\Omega_i (i = 1, 2, 3, 4)$, $\Omega_{i\Lambda} (i = 1, 2, 3, 4; \Lambda = L, R)$	introduced flux and current distribution regions
σ_{zz}	stress component
u, w	displacement components in the r and z directions, respectively
f	electromagnetic body force
$[N]$	shape function matrix
$[F]^e$	element nodal force vector
$[K]^e$	element stiffness matrix
A_e	element area

crack model are derived analytically for both zero-field cooling (ZFC) and field cooling (FC) activation processes. The expressions of them are much more complicated than the ones corresponding to the penny-shaped crack problem. The stress intensity factors (SIFs) at the different fronts are determined and numerically solved by virtue of the FEM, respectively. The effects of crack configuration, activation processes and applied fields on fracture properties of the annular crack are analyzed according to the maximum SIF criterion. The present work should be useful for the analysis and design of superconducting materials.

2. Problem statement and basic equations

As shown in Fig. 1(a), a superconducting cylinder contains an inner annular crack. The superconducting cylinder is placed in a magnetic field B_a oriented parallel to the cylinder axis (z axis). The inner and outer radii of the crack are a and b, respectively, and the radius of the cylinder is R. The coordinates in the cylindrical coordinate system are r, θ and z, respectively. Because the superconducting cylinder is infinite in the z direction, the effects of the demagnetization are neglected here.