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Exact analysis of radial vibration of functionally graded piezoelectric ring transducers resting on elastic foundation

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ABSTRACT

An exact solution is obtained to analyze the electromechanical characteristics of functionally graded piezoelectric ring transducers in radial vibration. The material properties of piezoelectric ring transducers are assumed to be graded in power-law distribution form along the radial direction. The material inhomogeneity index of mass density can be different from that of the elastic, piezoelectric and dielectric constants. Analytical solution is obtained in terms of Bessel functions. The characteristic equations governing the resonant and anti-resonant frequencies are presented. Numerical results are depicted graphically to illustrate the effect of material inhomogeneity index and elastic foundation stiffness on the radial vibration characteristics, such as resonant/anti-resonant frequencies, effective electromechanical coupling factor and model shape.

1. Introduction

Piezoelectric materials can induce voltage when subjected to external mechanical loadings, and vice versa, they can deform by applying external voltage. Due to this special electromechanical coupling effect, piezoelectric materials have been widely used in various smart systems and devices. For instance, the electro-acoustic transducers have been extensively used in electro-acoustics, hydroacoustics, and ultrasonics. The configurations of these transducers can be disk, circular ring, long cylinder, cylindrical shell etc. [1,2].

Many studies have been performed for the homogeneous piezoelectric transducers. Adelman et al. studied the axisymmetric vibrations of radially polarized piezoelectric cylinders [3]. Ramesh and Ebenezer analyzed the axisymmetric vibration of axially polarized piezoelectric rings [4]. Kim and Lee studied the radial vibration of piezoelectric cylindrical transducers with radial polarization [5]. Huang et al. obtained the electro-mechanical responses of a long piezoelectric tube subjected to periodic excitations [6]. Lin et al. investigated the radial vibration of the radially poled piezoelectric ceramic long tubes with arbitrary wall thickness [7]. Wang et al. studied the radial vibration of radially polarized piezoelectric rings [8].

Functionally graded materials (FGMs) can be used to improve the electro-mechanical behaviors of piezoelectric structures and systems. FGMs are one kind of new materials of which the material properties vary continuous in the space domain. The different distribution forms of the material properties will cause different interactions at the interior of the structures. Among them, the cases that the material properties are assumed to be power-law function of radial position have been widely considered [9–19]. Some works on piezoelectric composite cylinders have also been reported [20–24].

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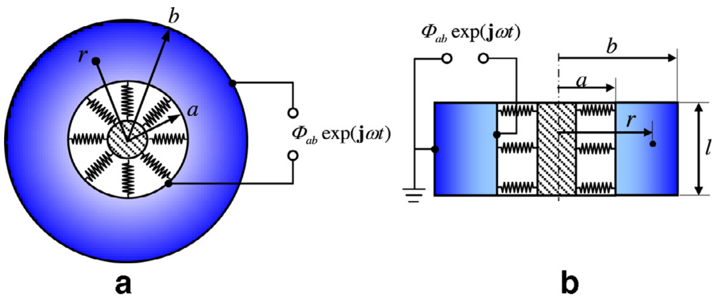

Fig. 1. Mechanical model of piezoelectric transducer resting on elastic foundation: (a) cross section, (b) longitudinal section.

In many cases, piezoelectric transducers should work at close to resonant or anti-resonant frequencies. As we known, the resonant and anti-resonant frequencies will be greatly affected by the boundary conditions. Generally, the mechanical boundary conditions can be traction free or fixed. Recently, some works have also been carried out for the multi-field coupling behavior of the smart structures resting on elastic foundations. Akbarzadeh and Chen [25] investigated the magnetoelectroelastic behavior of rotating cylinders resting on an elastic foundation subjected to hygrothermal loading. Akbarzadeh and Pasini [26] studied the hygrothermomagnetoelectromechanical behavior of multilayered and functionally graded cylinders resting on elastic foundations. Wang [27] investigated the piezothermoelastic behavior of multilayered pyroelectric cylindrical actuators with weakly conducting interfaces resting on elastic foundations. Jodaei [28] obtained the 3D elasticity solution for static analysis of functionally graded piezoelectric annular plates on elastic foundations. It is noted that only the steady-state problems were investigated in Refs. [25–28]. Yas [29] performed the three-dimensional free vibration analysis of functionally graded piezoelectric annular plates resting on elastic foundations. To the author's knowledge, the vibration behavior of the piezoelectric ring transducer resting on elastic foundations has not been investigated yet.

Piezoelectric ring transducers are a kind of newly developed high power acoustic transmitters [30]. Many applications require the piezoelectric transmitters operate in resonant frequency. In order to match the varying application demands, frequency tunable technique is important in ultrasonics engineering [31]. It was reported that the natural frequency can be tuned by adjusting the foundation stiffness [32]. Motivated by this work, we can also tune the natural frequency of the piezoelectric ring transducers by using the elastic foundation. In this investigation, the radial vibration behavior of functionally graded piezoelectric ring transducers resting on elastic foundation is studied. Based on the proposed solution, the design parameters, such as resonant/anti-resonant frequencies, effective electromechanical coupling factor and model shape, are analyzed.

2. Basic equations

Fig. 1 depicts the mechanical model of a piezoelectric ring transducer resting on elastic foundation. The inner radius, the outer radius and the height of the piezoelectric ring are denoted by *a, b* and *l*, respectively. In engineering practice, the height is much smaller than the outer radius. Assume the piezoelectric ring is traction free at the top and bottom surfaces. The state of stress in the piezoelectric ring can be treated as plane stress. Here we consider the radial vibration only. In the cylindrical coordinate system (r, θ, z), the mechanical displacement in radial direction and the electric potential can be expressed as $u_r = u_r(r, t)$ and $\Phi = \Phi(r, t)$, respectively. Here t is the time variable. Suppose the piezoelectric ring transducer is excited by a harmonic voltage $\Phi_{ab}\exp(\mathrm{j}\omega t)$. Here Φ_{ab} is the amplitude of voltage excitation and $\exp(\mathrm{j}\omega t)$ is the harmonic factor, ω is the angular frequency and j is the imaginary unit. For the sake of brevity, $\exp(\mathrm{j}\omega t)$ is dropped in the following analysis.

The equation of motion in radial direction is [3] as follows:

$$\frac{\mathrm{d}\sigma_{rr}}{\mathrm{d}r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \rho\omega^2 u_r = 0, \tag{1}$$

where σ_{rr} is the radial stress, $\sigma_{\theta\theta}$ is the hoop stress, and ρ is the mass density.

The charge equation of electrostatics is simplified as [3] follows:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rD_{rr}) = 0, \tag{2}$$

where D_{rr} is the electric displacement. The constitutive relations for radially polarized piezoelectric media are [33] as follows:

$$\begin{aligned}
\sigma_{\theta\theta} &= c_{11}\frac{u_r}{r} + c_{12}\varepsilon_{zz} + c_{13}\frac{\mathrm{d}u_r}{\mathrm{d}r} + e_{31}\frac{\mathrm{d}\Phi}{\mathrm{d}r},\\
\sigma_{zz} &= c_{12}\frac{u_r}{r} + c_{22}\varepsilon_{zz} + c_{23}\frac{\mathrm{d}u_r}{\mathrm{d}r} + e_{32}\frac{\mathrm{d}\Phi}{\mathrm{d}r},\\
\sigma_{rr} &= c_{13}\frac{u_r}{r} + c_{23}\varepsilon_{zz} + c_{33}\frac{\mathrm{d}u_r}{\mathrm{d}r} + e_{33}\frac{\mathrm{d}\Phi}{\mathrm{d}r},
\end{aligned} \tag{3}$$