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Consolidation analysis of transversely isotropic layered saturated soils in the Cartesian coordinate system by extended precise integration method

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ABSTRACT

A general solution for three-dimensional consolidation of transversely isotropic layered saturated soils is presented as an extension to the extended precise integration solution for consolidation in the cylindrical coordinate system. Starting with the governing equations of Biot's consolidation of transversely isotropic saturated soils in the Cartesian coordinate system, an ordinary differential matrix equation is deduced with the aid of Laplace–Fourier transforms. Based on the precise integration method for two-point boundary value problems, an extended precise integration solution of multilayered systems subjected to internal loads or dislocations is presented and then used to solve the above ordinary differential matrix equation, the actual solution in the physical domain is obtained by taking a numerical inversion. The feasibility of the proposed method is proved by three examples, and the influence of transverse isotropy of the soil skeleton on the consolidation behavior is analyzed.

1. Introduction

Biot's theory of consolidation [1], which considers the coupling between the soil skeleton and the pore fluid, is regarded as one of the most reasonable theory to study the time-dependent behavior of saturated soil. However, for the complexity of the porous elastic medium's governing equations, the solution based on the theory is difficult to derive. Up to now, some analytic theories have been presented to deal with this problem. For the plane strain and axisymmetric consolidation of a semi-infinite clay stratum, McNamee and Gibson [2,3] presented the analytic solutions by the introduction of two displacement functions, and their method was further extended to obtain the solutions of non-axisymmetric consolidation problems [4]. Considering the fact that most soils are naturally layered in character, some investigators shifted their focus on studying the consolidation problems of multilayered soils. The transfer matrix method [5–8] and the finite layer method [9–11] were usually proposed to study the consolidation of multilayered soils. However, the elements of the transfer matrices and stiffness matrices used in these methods usually contain positive exponential functions, which may lead to ill-conditioned matrices and computation overflow. The thin-layer or discrete layer approach [12,13] for cross-anisotropic or anisotropic materials may be an available way to solve consolidation problems of layered systems, but corresponding achievements have not been reported. The propagator matrix method with modified formulations [14,15] shows good stability and accuracy in numerical calculation for multilayered elastic media and has been applied frequently in geophysics to study dislocation problems [16]. Besides, an exact stiffness matrix

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method [17] whose elements of the stiffness matrix only contains negative exponential functions is proposed to calculate the quasi-statics consolidation problems of a multilayered poroelastic medium under circular loading. Recently, another analytical method named analytical layer-element method [18,19], which can also avoid the overflow in computation, was further developed to study the axisymmetric and non-axisymmetric consolidation of multilayered soils.

In view of the sedimentation process and the variation of orientation in different directions of the soil, it is necessary to consider the transversely isotropic properties in consolidation calculation. Compared with the Biot's consolidation problems of isotropic saturated soils, it is extremely difficult to deduce the explicit analytical solutions of transversely isotropic saturated soils for its more complex constitutive relation. On the other hand, the traditional numerical methods, such as the finite element method, are still time-consuming in dealing with the consolidation problems. Recently, by combining the integral transform method with the precise integration method, the authors proposed a new method [20] to solve the Biot's consolidation problems of transversely isotropic layered saturated media in the cylindrical coordinate system, and proved that the method possesses considerable efficiency and accuracy.

Based on the governing equation of Biot's consolidation of transversely isotropic layered saturated soils in the Cartesian coordinate system, the differential equations are derived in the transformed domain by applying the Laplace transform and Fourier transform to t and x (y), respectively. Extending the precise integration method, the precise integration solution of multilayered soils under the internal loading or dislocating is deduced. With the aid of the presented method, the consolidation solutions for transversely isotropic saturated soils are acquired by solving the differential equations in the transformed domain. The actual solutions in the physical domain are obtained by inverting the Laplace–Fourier transforms. Comparing with the solution presented in Ref. [20], the authors would like to demonstrate the innovation points of this paper as follows: (1) a more generalized solution for three-dimensional consolidation is presented as an extension to the original solution in the cylindrical coordinate system. (2) Both the internal loading and dislocating are considered in the improved precise integration method. (3) Sensitivity analysis of transverse isotropy of consolidating soils' skeleton is carried out by numerical examples.

2. Governing differential equations of Biot's consolidation

In the absence of body forces, the equilibrium equations of 3D elasticity can be expressed as:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}=0, \tag{1a}$$

$$\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{y}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}=0, \tag{1b}$$

$$\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{z}}{\partial z}=0, \tag{1c}$$

where σ_x, σ_z, σ_z are normal stress components in the x, y, and z directions, respectively; σ_{xz}, σ_{yz}, σ_{xy} denote shear stress components in the $x-z$, $y-z$ and $x-y$ planes, respectively.

According to the principle of effective stress, we have:

$$\boldsymbol{\sigma}_s=\boldsymbol{\sigma}'_s-\boldsymbol{\sigma}, \tag{2}$$

where $\boldsymbol{\sigma}_s=[\sigma_x,\sigma_y,\sigma_z,\sigma_{xz},\sigma_{yz},\sigma_{xy}]^{\mathrm{T}}$ and $\boldsymbol{\sigma}'_s=[\sigma'_x,\sigma'_y,\sigma'_z,\sigma'_{xz},\sigma'_{yz},\sigma'_{xy}]^{\mathrm{T}}$ denote the total stress vector and the effective stress vector, respectively; $\boldsymbol{\sigma}=[\sigma,\sigma,\sigma,0,0,0]^{\mathrm{T}}$ is the excess pore fluid pressure vector (pressure as positive).

The constitutive equations of transversely isotropic medium can be expressed as follows:

$$\boldsymbol{\sigma}'_s=\begin{bmatrix} c_{11} & c_{12} & c_{13} & \\ c_{12} & c_{11} & c_{13} & \mathbf{0}_{3\times3} \\ c_{13} & c_{13} & c_{33} & \\ & \mathbf{0}_{3\times3} & & \mathbf{c}_{3\times3}\end{bmatrix}\boldsymbol{\varepsilon}_s, \tag{3}$$

in which,

$$\mathbf{c}_{3\times3}=\begin{bmatrix} c_{44} & 0 & 0 \\ 0 & c_{44} & 0 \\ 0 & 0 & (c_{11}-c_{12})/2\end{bmatrix}, \tag{4}$$

where $c_{11}=\lambda\zeta(1-\zeta\nu_{vh}^2)$, $c_{12}=\lambda\zeta(\nu_h+\zeta\nu_{vh}^2)$, $c_{13}=\lambda\zeta\nu_{vh}(1+\nu_h)$, $c_{33}=\lambda(1-\nu_h^2)$, $c_{44}=G_v$, in which, $\lambda=E_v/[(1+\nu_h)(1-\nu_h-2\zeta\nu_{vh}^2)]$, $\zeta=E_h/E_v$; E_h, E_v and G_v are the horizontal Young's modulus, vertical Young's modulus and vertical shear modulus, respectively; ν_h and ν_{vh} denote Poisson's ratios characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallel and normally to the plane, respectively; $\boldsymbol{\varepsilon}_s=[\frac{\partial u_x}{\partial x},\frac{\partial u_y}{\partial y},\frac{\partial u_z}{\partial z},\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x},\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y},\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}]^{\mathrm{T}}$, u_x, u_y and u_z are the displacement components in the x, y and z directions, respectively.