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On reconstruction of thermalphysic characteristics of functionally graded hollow cylinder

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An inverse coefficient problem of thermal conductivity for a functionally graded hollow cylinder is considered. After applying the Laplace transform, the direct thermal conductivity problem is solved by using two methods: (1) based on a reduction to the Fredholm integral equation of the 2nd kind; (2) by means of the Galerkin method. A comparison of the direct problem solving techniques is provided. The nonlinear inverse problem is solved on the basis of an iterative process; at every step of the latter the linear Fredholm integral equation of the 1st kind is solved. Results of the computational experiments on a reconstruction of variation laws of thermal conductivity and specific volumetric heat capacity are presented.

1. Introduction

Investigation of heat conduction in bodies of cylindrical shape is of great importance in many up-to-date fields of science and technology including hardening processes, fuel cells, electrochemical reactors fabrication, and high density microelectronics. For quantitative calculations of heat conduction processes, it is required knowing thermophysical characteristics of materials. When simulating, they often assume that material is homogeneous, therefore its thermophysical properties are described by a set of physical constants that are determined from simple macro- experiments.

However, in recent years, functionally graded materials (FGM) have been increasingly becoming embedded in various fields of technology acting as an alternative to laminates. FGM is a composite possessing variable physical properties which, as opposed to laminates, avoid jumps of thermalphysic properties through the interfacial area [1]. But FGM manufacture is a complicated processing procedure. Due to multistage manufacturing operations, there can be a deviation from the established norms in the end item. In case of inhomogeneous bodies, direct measurements of thermophysical characteristics are impossible as far as they represent some functions of coordinates. Therefore, an efficient technique of FGM control quality on the basis of solving coefficient inverse problems (IP) of heat conduction is required after fabrication [2].

There is a plenty of papers devoted to a reconstruction of thermophysical properties of inhomogeneous materials [3–12]. Many authors reduce coefficient IPs to a solving of corresponding extremum problems [3–10]. To do it, they introduce non-quadratic residual functional which is minimized in a finite-dimensional subspace by means of gradient methods. In this way, in

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[10] they considered a problem of identification of thermal conductivity coefficient of endless hollow cylinder under stationary temperature field. To solve the problem, the authors used the conjugate gradient method and the discrepancy principle. As an input data, measured temperature at the inner points of the cylinder was taken. The identification was carried out only in a class of exponential functions. It worth noting that usage of minimization gradient methods requires sufficient computational time and suffers from other shortcomings: strong influence of initial approximation selection on iterative process convergence, and efficiency function requirements. At present, alternate methods of coefficient IP solving is still to be searched.

In [11] a reconstruction of thermal conductivity coefficient of wave heat conduction equation for a layer is considered. Inhomogeneous layer is divided into horizontal homogeneous layers, and the direct problem is solved using finite-difference method. To solve an inverse problem, the Newtonian iterational algorithm is applied. Corrections of the function to reconstruct are found by a solving of Fredholm's integral equation of the 1st kind obtained by the linearization method based on the orthogonality principle.

In the papers [12–15] a reconstruction of thermomechanical characteristics of an inhomogeneous rod is considered. In these papers, a solving of direct problems is reduced to a single Fredholm's integral equation or a system of Fredholm's integral equations of the 2nd kind. Nonlinear IPs are solved by means of building an iterative procedure, at every step of which a linear Fredholm's integral equations of the 1st kind is solved. Linearization is occurred either by using orthogonality principle [12], or by using weak statement of direct problem [13], or on the basis of the generalized reciprocal relation [14–21].

The present article deals with identification of thermal conductivity coefficient and specific volumetric heat capacity of an endless hollow cylinder. The direct problem of nonstationary thermal conductivity for a FGM cylinder after the Laplace transformation is solved in two ways: (1) by means of reduction to the Fredholm integral equation of the 2nd kind, as it was done for a rod [12]; (2) by the Galerkin method of weighted residuals. The inversion of the obtained solutions' transforms is occurred on the basis of the theory of residues, as in [15]. The techniques of solving the direct problems are compared with each other. To solve the IP for a cylinder, we expanded the aproaches proposed before to solve the IP for a rod [12–15]. A temperature measured in some time interval in a point of cylinder's external surface is taken as the input data in the IP. On the basis of the weak statement of the direct problem, the IP is reduced to stepwise solving of the Fredholm integral equation of the 1st kind. A series of computational experiments with exact and noised input data is conducted. The recommendations on practical employment of the approach proposed are given.

2. Statement of heat conduction coefficient IP for endless hollow cylinder

Consider a problem on heat conduction in inhomogeneous endless hollow cylinder, assuming that at the inner surface $r = a$ a zero temperature is maintained, and at the outer surface $r = b$ there is a constant heat flow $q = q_0 H(t)$. In this case, a temperature represents function of radial coordinate and time $T = T(r, t)$, and thermophysical characteristics are functions of radial coordinate only. The initial-boundary value problem in cylindrical coordinates takes form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(r) r \frac{\partial T}{\partial r}\right) = c(r)\frac{\partial T}{\partial t}, \quad a \le r \le b,\ t > 0, \tag{1}$$

$$T(a, t) = 0, \quad -k(b)\frac{\partial T}{\partial r}\bigg|_{r=b} = q_0 H(t), \tag{2}$$

$$T(r, 0) = 0. \tag{3}$$

Let us use dimensionless parameters and functions in (1)–(3), denoting: $z = \frac{r-a}{b-a}$, $z_0 = \frac{a}{b-a}$, $\bar{k}(z) = \frac{k(r)}{k_0}$, $\bar{c}(z) = \frac{c(r)}{c_0}$, $\tau = \frac{k_0 t}{c_0 (b-a)^2}$, $W(z, \tau) = \frac{k_0 T}{q_0 (b-a)}$, $k_0 = \max_{r\in[a,b]} k(r)$, $c_0 = \max_{r\in[a,b]} c(r)$. After it, the initial-boundary value problem (1)–(3) will take form:

$$\frac{1}{z+z_0}\frac{\partial}{\partial z}\left(\bar{k}(z)(z+z_0)\frac{\partial W}{\partial z}\right) = \bar{c}(z)\frac{\partial W}{\partial \tau}, \quad 0 \le z \le 1,\ \tau > 0, \tag{4}$$

$$W(0, \tau) = 0, \quad -\bar{k}(1)\frac{\partial W}{\partial z}\bigg|_{z=1} = H(\tau), \tag{5}$$

$$W(z, 0) = 0. \tag{6}$$

The direct problem of heat conduction is to determine the function $W(z, \tau)$ from (4)to (6) while the thermophysic characteristics, $\bar{c}(z)$, $\bar{k}(z)$ are known.

In the IP, it is required to simultaneously define two thermophysic characteristics of a cylinder $\bar{c}(z)$ and $\bar{k}(z)$. To investigate problems that arise when solving inverse coefficient problems of heat conduction for a cylinder, it is natural to launch an investigation in the framework of some simplified statements when only one characteristic is unknown, while the other one is assumed to be a constant.

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