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Objective Bayesian analysis accelerated degradation test based on Wiener process models[☆]

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ABSTRACT

Constant-stress accelerated degradation test (CSADT) as an effective model is widely used in assessing product reliability when measurements of degradation leading to failure can be observed. We model the degradation process as a Wiener process. In this paper, we make inferences about the parameters of the CSADT using an objective Bayesian method. The non-informative priors (Jefferys prior and two reference priors) are derived, and we show that their posterior distributions are proper. Since the posterior distributions are very complicated, Gibbs sampling algorithms for the Bayesian inference based on the Jefferys prior and two reference priors are proposed. Some simulation studies are conducted to show the effectiveness of the objective Bayesian analysis. Finally, we apply the objective Bayesian method to a real data set and estimate the mean-time-to-failure (MTTF) under use condition.

1. Introduction

With today's high technology, most of the manufactured products are highly reliable. Therefore, it is difficult to observe failure times under normal operating conditions. Accelerated life tests (ALTs) have been employed in industry to overcome such difficulties. However, for many products often few or no failures can be observed in a short life testing time even under accelerated life tests. As a result, it is difficult to estimate the reliability of those high-reliability products. In such a case, if there exists a quality characteristic related to reliability, which degrades over time (also called degradation paths), an alternative approach is to collect the degradation data at higher levels of stress and then predict the lifetime distribution of the products at a normal use-stress level. Such an experiment is called an accelerated degradation test (ADT). See the details in [1,2].

ADTs can be broadly classified into constant-stress ADT (CSADT), step-stress ADT (SSADT) and progressive-stress ADT (PSADT) according to different stress loading methods. The degradation model describes the behavior of the degradation characteristic as a function of time. The typical and most used degradation model is the mixed effects models. The mixed effects models consists of an actual degradation path involving fixed and random parameters and an error term. The fixed parameters are common to a population of units, whereas the random parameters represent the unit-to-unit variability. The error term in a mixed effects model usually represents the measurement error assumed to be independent over time. General references for this approach

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are [3,4]. However, if the correlation among degradation measurements over time is significant, an alternative stochastic process (SP) model can be naturally taken into consideration, see detail in [5,6]. Based on the assumption of additive accumulations of degradation, the Wiener processes [7,8] and the gamma processes [9,10] have been well exploited. In addition to the Wiener and the Gamma processes, a useful model for degradation is the inverse Gaussian process [11,12].

In practice, the degradation process may be affected by many factors (also called covariates) such as temperature, humidity, etc. Different approaches to incorporate covariates in the Wiener process model can be found in (see for example [13–15]. In particular, Ye et al. [16] review all kinds of the stochastic degradation models with several accelerating variables.

In all the above works, the research methods for degradation models are the frequentist approaches or the subjective Bayesian method. However, the objective Bayesian method has many advantages in the statistical analysis, see [17]. The most familiar element of the objective Bayesian approaches is the use of noninformative prior distributions. The Jeffreys prior and the reference prior are the two most used noninformative priors. For more details, see [18–20].

Xu and Tang [21] used the objective Bayesian method for linear degradation models. However, the accelerated stress and stochastic process are not considered. We consider objective Bayesian analysis of accelerated degradation data, utilizing non-informative prior distribution for the unknown parameters of Wiener process. The main motivation for this paper is that the commonly used noninformative priors can fail to yield proper posterior distributions. The primary methodological goal of this article is to provide noninformative prior distributions for ADTs that do result in proper posterior distributions and that have additional desirable properties.

The rest of this article is organized as follows. SP degradation model and related works are reviewed in Section 2. The Fisher information matrices under the original parametrization and reparametrization are given in Section 3. In Section 4, we derive two possible reference priors, and show that the posterior distributions are proper based on these priors. In Section 5, we analyse the property of posterior distribution under the reference priors, and three Gibbs sampling are proposed to estimate the parameters. In Section 6, two numerical examples are studied to show the effectiveness of the methods. A real data set from [15] is analyzed in Section 7. Finally, conclusion is made in Section 8.

2. Stochastic process degradation models

2.1. Wiener degradation process models

Degradation is modeled as a Wiener process. Let $B(t)$ be the standard Wiener process characterized by the following:

- $B(0) = 0$.
- $B(t|t \geq 0)$ has stationary independent increments, and each increment, $\triangle B = B(t + \triangle t) - B(t)$, follows a normal distribution with mean 0 and variance $\triangle t$.

Let $Y(t)$ be the degradation characteristic at time t and define $y(t) = Y(t) - Y(0)$. In this paper, $y(t)$ is modeled as,

$$y(t) = \delta B(t) + \eta t,$$

where $\eta > 0$ and $\delta > 0$. That is, $y(t)$ is assumed to follow a Wiener process with drift η and diffusion constant δ^2.

The following assumptions A1–A5 are considered.

A1: Constant stress loading is adopted at each stress level $S_i, i = 1, \ldots, r$, and n_i units are allocated to each stress level.

A2: The maximum and use stress levels are pre-specified as S_M and S_0, respectively. Under any constant stress level $S_i, i = 0, 1, \ldots, r$, the degradation characteristic $y_{ij}(t)$ of the jth unit ($j = 1, 2, \ldots, n_i$) follows a Wiener process with drift $\eta(S_i)$ and diffusion constant δ^2.

A3: A unit is assumed to fail when the degradation characteristic $y_{ij}(t)$ becomes greater than the critical value ω.

A4: The lifetime at a lower stress level tends to be longer than that at a higher stress level. The relationship between the mean (median, or percentile) lifetime of the products and stress are given, usually based on engineering background. Stress variables that can be used for CSADT include temperature, voltage, humidity, mechanical load, vibration, etc. Three such most commonly used relationships are as follows (see [1]):

- Arrhenius model: $\ln(\Theta) = \gamma_0 + \frac{\gamma_1}{\nu}$, where ν is the absolute temperature,
- Inverse power model: $\ln(\Theta) = \gamma_0 + \gamma_1[-\ln(\nu)]$, where ν is the voltage,
- Exponential model: $\ln(\Theta) = \gamma_0 + \gamma_1 \nu$, where ν is a weathering variable.

Thus $\ln(\Theta)$ is a linear function of the transformed stress $\varphi = \frac{1}{\nu}, -\ln(\nu)$ or ν for the above three models. The lifetime Θ is usually taken to be a specified percentile of the life distribution. For the sake of generosity, we assume in this paper that the relationship between the mean time to failure ($MTTF_i$) and the stress level $S_i, i = 0, 1, \ldots r$, has the form,

$$\ln(MTTF_i) = \gamma_0 + \gamma_1 \varphi(S_i), \tag{1}$$

where γ_0 and γ_1 are unknown parameters to be estimated.

A5: Let n_{ij} be the number of measurements for the jth unit at the stress level S_i. It is assumed that $n_{ij} = m$ for all i and j. The measurement times ($t_{ijk}, k = 1, 2, \ldots, m$) and maximum test duration (t_{Mij}) for the jth test unit at the stress levels S_i are pre-determined. In particular, it is assumed that $t_{Mij} = t_M$, $t_{ijk} - t_{ijk-1} = z$ and $t_{ijm} = t_M$ for all i, j and k. That is $t_{ijm} = mz$.