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Mixed convection hydromagnetic flow in a rotating channel with Hall and wall conductance effects

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ABSTRACT

Mixed convection hydromagnetic flow of a viscous, incompressible, electrically and thermally conducting fluid in a rotating channel taking Hall current into account is studied. Fluid flow within the channel is induced due to an applied pressure gradient acting along the longitudinal axis of the plates of the channel. Exact solution of the governing equations is obtained in closed form. Expressions for the shear stress and critical Grashof number at the plates of the channel due to primary and secondary flows and mass flow rates in the primary and secondary flow directions are also derived. Asymptotic behavior of the solution for fluid velocity and induced magnetic field is analyzed for large values of rotation parameter K^2 and magnetic parameter M^2 to gain some physical insight into flow pattern. Heat transfer characteristics of fluid flow are considered taking viscous and Joule dissipations into account. Numerical solution of energy equation and numerical values of rate of heat transfer at the plates of the channel are computed with the help of MATLAB software. The numerical values of fluid velocity, induced magnetic field and fluid temperature are displayed graphically versus channel width variable η for various values of pertinent flow parameters whereas that of shear stress and critical Grashof number at the plates of the channel due to primary and secondary flows, mass flow rates in the primary and secondary flow directions and rate of heat transfer at the plates of the channel are presented in tabular form for various values of pertinent flow parameters.

1. Introduction

Theoretical/experimental investigation of the problems of hydromagnetic flow of an electrically conducting fluid permeated by a magnetic field with heat transfer is of considerable interest because it has several practical applications in plasma aerodynamics [1], MHD power generator [2], nuclear engineering [3], astrophysical and geophysical fluid dynamics [4,5] and manufacturing process in industry [6, 7]. Problems of magnetohydrodynamic heat transfer may be roughly divided into two groups. In group one, heating is an incidental byproduct of electromagnetic fields. This group includes devices like MHD generators, MHD accelerators and to a lesser degree MHD pumps and flow meters. These are broadly mentioned as channel or duct flows. In group two, the primary use of electromagnetic fields is to control heat transfer. This group contains free convection flows and aerodynamic heating where geometric configurations are varied. It is worthy to note that, in MHD heat transfer problems, the usual Reynolds analogy between skin friction and heat transfer, as in non-conducting fluid, does not hold in general. This is due to the fact that, in addition to the viscous dissipation, there is a Joule dissipation of heat caused by flow of electric current

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in the fluid. Hydromagnetic channel flow with heat transfer due to forced convection is studied by Yen [8], Jagadeesan [9] and Soundalgekar [10] under different conditions. The effects of thermal buoyancy force on MHD forced convection flow are of practical interest because fluids with low Prandtl number are electrically conducting and are more sensitive to the gravitational field than that with high Prandtl number. Hence the interplay of thermal buoyancy force with the electromagnetic forces determines the ultimate behavior of an electrically conducting fluid with low Prandtl number in the presence of magnetic field. Taking into consideration this fact Gill and Casal [11], Gupta [12], Das [13], Jana [14], Singh [15], Mazumder et al. [16], Datta and Jana [17], Kant [18], Seth and Ghosh [19], Ghosh and Nandi [20], Ghosh et al. [21] and Guria et al. [22] studied MHD mixed convection flow of a viscous, incompressible and electrical fluid within a parallel plate channel in the presence of magnetic field by considering different aspects of the fluid flow problem.

Investigation of the problems of hydromagnetic flow of an electrically conducting fluid in a rotating environment assumes considerable importance due to the occurrence of various natural phenomena which are generated by the action of Coriolis and magnetic forces. An order of magnitude analysis shows that the effects of Coriolis force are more significant than that of inertial and viscous forces in the magnetohydrodynamic equations of motion in a rotating system whereas Coriolis and electromagnetic forces are comparable in magnitude. Investigation of such fluid flow problems may find application in several areas of astrophysics, geophysics and engineering viz. maintenance and secular variations of terrestrial magnetic field due to motion of Earth's liquid core, internal rotation rate of sun, planetary and solar dynamo problems, structure of rotating magnetic stars, MHD Ekman pumping, turbo machines, rotating MHD generators, rotating drum type separators in closed cycle two phase MHD generator flow, etc. Combined effects of Coriolis and magnetic forces on hydromagnetic mixed convection flow of a viscous, incompressible and electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field is studied by Mohan [23], Sarojamma and Krishna [24], Shivaprasad et al. [25], Prasad Rao et al. [26], Ghosh and Bhattacharjee [27], Seth and Singh [28], Seth and Ansari [29], Seth et al. [30] and Singh and Pathak [31] to analyze different aspects of this problem. It was pointed out by Gebhart [32] that the effect of viscous dissipation plays a vital role in natural convection in various devices that are subjected to large deceleration, or which operate at high rotating speeds and also in strong gravitational field processes on large scales (on large planets) and geological process. Under such situations if an electrically conducting fluid flows in the presence of magnetic field, the effects of viscous and Joule dissipations become significant because it act as volumetric heat source. Keeping in mind the importance of such study, Anjali Devi and Ganga [33] considered the effects of viscous and Joule dissipations on hydromagnetic natural convection heat and mass transfer flow past a stretching porous surface embedded in a porous medium. Seth and Singh [28] and Seth et al. [30] investigated combined free and forced convection MHD flow in a rotating channel with perfectly conducting and arbitrary conducting walls respectively taking viscous and Joule dissipations into account.

When an ionized fluid with low density is subjected to a strong magnetic field then the electrical conductivity normal to the magnetic field is lowered because of free spiraling of electrons and ions about the magnetic lines of force just before the collision and a current, namely, Hall current is thereby generated which is mutually perpendicular to electric and magnetic fields (Sutton and Sherman [34]). Most important characteristic of Hall current is that it induces secondary flow in the flow-field which is also characteristics of Coriolis force. Therefore, it is of practical interest to study the combined effects of Hall current and rotation on such fluid flow problems which may find applications in MHD power generation, nuclear power reactors and underground energy storage system and in several areas of astrophysics and geophysics. Sivaprasad et al. [25] studied Hall effects on MHD free and forced convection flow in porous rotating channel whereas Singh and Pathak [31] considered the effects of Hall current and rotation on mixed convection oscillatory MHD flow through a porous medium filled in a vertical channel in the presence of thermal radiation. Shivprasad et al. [25] and Singh and Pathak [31] have neglected induced magnetic field produced by fluid motion. Due to this reason Joule dissipation is not taken into account in their study. Abo-Eldahab and Aziz [35] studied the effects of viscous and Joule dissipations on MHD free convection flow past a semi-infinite vertical flat plate in the presence of Hall current and ion slip. Seth and Ansari [29] extended the problem considered by Seth and Singh [28] to include the effects of Hall current.

Present investigation deals with study of steady hydromagnetic mixed convection flow of a viscous, incompressible, electrically and thermally conducting fluid in a rotating channel with arbitrary conducting walls taking Hall current into account. Heat transfer characteristics of the flow are analyzed taking viscous and Joule dissipations into account. Lower wall of the channel is considered of finite thickness d_1 and electrical conductivity σ_1 whereas upper wall of the channel is considered of finite thickness d_2 and electrical conductivity σ_2. To solve this problem, we have derived boundary conditions for magnetic field taking Hall current into account. The numerical values of fluid velocity, induced magnetic field and fluid temperature are displayed graphically versus channel width variable η for various values of pertinent flow parameters whereas that of shear stress and critical Grashof number at the plates of the channel due to primary and secondary flows, mass flow rates in the primary and secondary flow directions and rate of heat transfer at the plates of the channel are presented in tabular form for various values of pertinent flow parameters. Numerical values of above physical quantities are also obtained when wall conductance ratios ϕ_1 and ϕ_2 are zero (i.e. when the walls are non-conducting) and when ϕ_1 and $\phi_2 \to \infty$ (i.e. when the walls are perfectly conducting) which are the particular cases of our problem. Therefore, the problem under consideration represents the most general case and the problem investigated by Seth and Ansari [29] is a special case of the present one.

2. Mathematical analysis

Consider flow of a viscous, incompressible, electrically and thermally conducting fluid between two infinite parallel arbitrary conducting plates $z = 0$ and $z = L$ in the presence of a uniform transverse magnetic field H_0 which is applied parallel to z-axis.